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A Rydberg-Bohr-de Broglie Algebraic Model of Hydrogen Atoms

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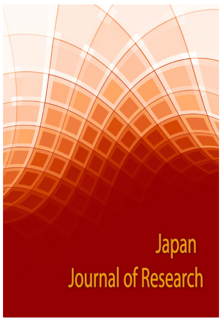

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Abstract

Bohr ignored the electromagnetic interaction of moving charges in atoms and only considered Coulomb forces, thus encountering half frequency difficulties. This article considers the Lorentz force to make the classical Bohr model consistent with modern quantum theory, uses de Broglie's standing wave principle instead of Bohr's quantization assumption, derives the energy level formula for hydrogen atoms, and explains the radiation mechanism of hydrogen atomic spectra. The hydrogen atom in a spectral tube emits a photon wave train during one free path, and the wavelength of the photon is the eigen wavelength of its wave train. The head wavelength of the wave train is four thirds of its eigen wavelength, and the tail wavelength of the wave train is two-thirds of its eigen wavelength.

Introduction

Since the papers of Bohr (1913), Sommerfeld (1916) and de Broglie (1923) were published, the study of atomic structure and quantum theory has entered a new era. However, a century later, the radiation mechanism and details for atomic spectrum are still missing in the existing theory. [1]

In 2020, Horodenko of Clemson University pointed out that Demkov and Ostrovsky (D-O) "believed that, in spite of the indistinguishability of electrons, Bohr's (circular orbits) and, later on, Sommerfeld's (elliptical orbits) treatment of hydrogen atom (Sommerfeld, 1934), is essential for obtaining the discrete spectrum of multielectron atom since the semiclassical-classical methods of treatment of the spectral problem should be associated with closed orbits" [1].

In measurements of "the absorption spectrum of hydrogen atoms to levels near the ionization threshold in a strong magnetic field", Haggerty and Delos fined that "the absorption rate has sinusoidal fluctuations which are correlated with closed classical orbits of the electron"[2]. It can be shown that each closed classical orbit of an electron generates a peak in a recurrence spectrum at the action of the orbit. Consequently, the recurrence spectrum provides a quantum picture of classical behavior [3]. "circular states have found applications in cavity quantum electro-dynamics, metrology, collisions, and intense radiation field dynamics"[4]. Closed orbitals are necessary to obtain discrete spectra and the Bohr model has laid a solid foundation for the study of closed orbits. This article was completed based on the preprint in 2021 [5].

Rydberg 's empirical formula

As early as the 1850s, people found four lines of hydrogen in the visible spectrum of sunlight: H_α, H_β, H_γ and H_δ. In 1884, Balmer, a middle school teacher in Switzerland, proposed a simple formula for calculating the wavelengths of these spectral lines [6].

$$\lambda = B_a \frac{n^2}{n^2-4} \qquad (1)$$

where n is any positive integer greater than 2, and $B_a = 3645.6\times10^{-10}$ m.

In 1890, Johannes Robert Rydberg of Sweden independently put forward a more general formula for the spectrum of hydrogen atom [7]

$$\tilde{\nu} = \mathrm{R}_y\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \qquad (2)$$

where $\tilde{\nu}$ is the wave number of the spectral line (cm^{-1}), n_1 and n_2 are positive integers, and $n_1 < n_2$; R_y is the Rydberg constant, and the determined value in 1987 is $R_y = 109737.31573(3)\ cm^{-1}$ [6] . If $n_1 = 2$, and let $n_2 = n$, the formula (2) can be simplified as Balmer formula (1); If $n_1 = 1$, and let $n_2 = n$, we obtain the Lyman series formula.

With the speed of light c, let $c\tilde{\nu} = \nu$ and $cR_y = \nu_R$, formula (2) can be rewritten as

$$\nu = \nu_\mathrm{R}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right); \quad \text{or} \quad \nu = \frac{\nu_\mathrm{R}}{n_1^2}-\frac{\nu_\mathrm{R}}{n_2^2} \qquad (3)$$

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Look at both sides of equation (3). The left, the ν is the frequency of the radiation photon. The right, the vR should be a fundamental frequency of the atomic vibration

$$v_R = cR_y = \ 2.99792458\times10^8 m/s\ \times\ 1.09737316\times10^7 m^{-1} = 3.28985441\times10^{15} s^{-1}. \quad (4)$$

Here, we call v_R the Rydberg frequency.

Bohr model and his half frequency problem

In 1913 Bohr assumed that electrons in atoms move in some stable orbits, different orbits belong to different energy levels, and that electrons moving in these stable orbits neither absorb nor radiate photons; Electrons absorb or radiate photons only when they jump between different levels [8]. Bohr neglected “the electromagnetic forces due to the motion of the electrons”. He first dealt with the hydrogen atom, and the lowest energy orbit was called the ground state.

When an object moves in a circle with velocity (υ), the centrifugal force (F_X) it is subjected to is

$$F\ \ = \frac{m\upsilon}{r} \qquad (5)$$

where m is the electron mass, r the radius of the circle. In a hydrogen atom, according to Newton's third law, Bohr believed that the Coulomb force F_C between electron and proton provides the centripetal force for the electron moving in circle

$$F_C = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad (6)$$

where e is the charge of proton or electron, ε_0 is the dielectric constant in vacuum. At that time, Bohr thought that in an equilibrium system of hydrogen atom , $F_C = F_{X.}$ Then, by combining formula (5) and formula (6), the following formula is got

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m\upsilon^2}{r}; \quad \text{or} \quad \frac{e^2}{4\pi\varepsilon_0 r} = m\upsilon^2 \qquad (7)$$

For the hydrogen atom in the ground state, all the physical quantities in formula (7) are constants.

Bohr used quantization condition

$$m\ \ r = \frac{h}{2} \qquad (8)$$

where h is Planck constant. By formulae (7) , we have

$$\frac{e^2}{4\pi\varepsilon_0} = m\upsilon r\cdot\upsilon \qquad (9)$$

Substituting formula (8) into formula (9), we get

$$\upsilon = \frac{e^2}{2h\varepsilon_0} \qquad (10)$$

According to both the formulae (8) and (7) , Bohr got his frequency formula

$$\nu_B = \frac{4\pi^2 m e^2 E_B{}^2}{\tau^3 h^3} \qquad (11)$$

where τ = 1, $E_B = e$, and Bohr used ‘ω’ for ν_B. By introduced the experimental values of following constants at that time

$$e = 4.7\times10^{-10}, \quad \frac{e}{m} = 5.31\times10^{17}, \quad h = 6.5\times10^{-27}, \qquad (12)$$

Bohr got

$$2a = 1.1\times10^{-8}\,\text{cm}, \quad \nu_B = 6.2\times10^{15}\,\text{s}^{-1}, \quad w = 13\ \text{eV} \qquad (13)$$

where a is Bohr’s radius, and w the ground state energy of hydrogen atoms. At the same time, he also derived and calculated the Rydberg frequency formula and the observed value

$$\nu_R = \frac{2\pi^2 m e^4}{h^3} = 3.1\times10^{15}$$

and the observed value 3.290×10^{15} (14)

Comparing formulas (14) with (11), the frequency ν_B calculated by Bohr must add a 1/2 factor to be consistent with the empirical Balmer formula [8, 9]

$$f(\tau) = \frac{\tau}{2} \qquad (15)$$

The 1/2 factor involves two different formulas represented by the same physical constants, which is obviously not a measurement error issue and later referred to as the "half frequency problem". In “§ 2 The Emission of Line spectrum” of his first paper, Bohr spent length of nearly 5 pages attempting to solve and explain this "half frequency problem".

After introducing this “half frequency” factor, Bohr can deduce Balmer formula and Rydberg constant from the perspective of the original constant (such as electron charge). Therefore, Bohr has great confidence in his classical theory.

He insisted that quantum mechanics should be a theory as little as possible divorced from classical mechanics [10]. According to the missing radiation mechanism in quantum theory, Bohr tried to restore the relationship between radiation and electron motion in quantum theory similar to classical radiation theory. However, Bohr did not succeed in his life. Not only did he not put forward new hypotheses about the mechanism of radiation, but after 1924 he began to avoid the original hypotheses [9]. He can only use the limiting case where $n\rightarrow\infty$, n-1→n, under high quantum number to weaken the "half frequency" difference, which is the correspondence principle he later advocated. Strictly speaking his correspondence principle is exact only in the limit of large quantum numbers, and cannot be extended to small quantum numbers. Darrigol writes, ‘Confronted with this paradoxical appearance of Bohr’s work, many physicists and historians have renounced the project of giving a rational account of it. In their opinion, Bohr’s success owed much to an unusual tolerance for contradiction . . .’[11]

Dr. Peter Vickers from the University of Pittsburgh in the United States said,“‘successes suggest truth’, regardless of any failures the theory has. Failures can be explained by the fact that a theory does contain false claims” [12].

So, behind the success of the Bohr model, what other false claims led to his half frequency problem? Bohr's derivation starts with the centrifugal force acting on an electron being equal to the centripetal force, which is, formula (5) = formula (6). The centrifugal force represented by formula (5) is not incorrect. If there is an error, it may be due to the centripetal force. As is well known, the atomic structure is very different from the planetary structure of the solar system. The centripetal force acting on the orbit of a planet is only gravitational force, which does not depend on the motion or stillness of celestial bodies; And atoms are composed of charged particles, in addition to the Coulombic attraction between stationary opposite charges, orbital charges also generate a magnetic field, and moving charges in the magnetic field are also affected by the Lorentz force. The Lorentz force also generates centripetal forces on electrons. Only considering the Coulomb force and ignoring the Lorentz force, the centripetal force of the Bohr model is incomplete.

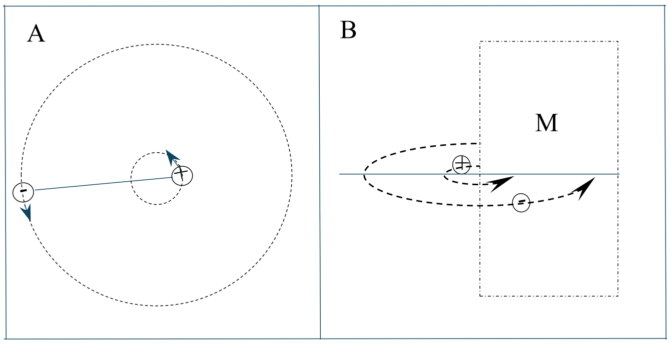

Figure 1. A) In a hydrogen atom, an electron and a protons move around their common center of mass; B) The number of negative charges passing through half plane M per unit time is equal to the number of positive charges.

Lorentz force in hydrogen atom

We analyze the generation of orbital magnetic fields in hydrogen atoms. As shown in Fig.-1A, an electron and a proton in hydrogen atoms rotate around their common center of mass. As shown in Fig.-1B, the number of positive and negative charges passing through a half plane M in a hydrogen atom per unit time is equal, resulting in two circular currents of equal magnitude and opposite directions. Two circular currents form two coupled magnetic fields with opposite polarity. These two coupled magnetic fields depend on the orbital motion of charges.

The Coulomb force $\boldsymbol{F}_{\mathrm{C}}$ and the Lorentz force $\boldsymbol{F}_{\mathrm{L}}$ are two independent forces. Both they should exist in the hydrogen atom and apply centripetal force to electron.

If the electron charge is q, the electron velocity is υ, the electric field is $\boldsymbol{E}_{\mathrm{C}}$ and the magnetic field is $\boldsymbol{B}$, the centripetal force $\boldsymbol{F}$ on the electron can be given by the following Lorentz equation

$$F = F_{\mathrm{C}} + F_{\mathrm{L}} = qE_{\mathrm{C}} + q(\upsilon \times B) \qquad (16)$$

where

$$F_{\mathrm{C}} = qE_{\mathrm{C}},\ F_{\mathrm{L}} = q(\upsilon \times B) \qquad (17)$$

The Coulomb force $\boldsymbol{F}_{\mathrm{C}}$ requires the presence of nuclear charges, but does not require the magnetic fields and the movement of charges; Lorentz force $\boldsymbol{F}_{\mathrm{L}}$ requires the magnetic fields and the movement of charges, but does not require nuclear charges. If Coulomb force and Lorentz force exist separately in the circular motion of electrons

$$F_{\mathrm{C}} = \frac{m\upsilon^2}{r}; \qquad F_{\mathrm{L}} = \frac{m\upsilon^2}{r} \qquad (18)$$

If Coulomb force and Lorentz force coexist in the circular motion of an electron, the centripetal force $\boldsymbol{F}$ on the electron is

$$F = F_{\mathrm{C}} + F_{\mathrm{L}} = 2F_{\mathrm{C}} = 2F_{L} = \frac{m\upsilon^2}{r}; \qquad (19)$$

By formulae (6) and (19), we obtain

$$\frac{e^2}{2\pi\varepsilon_0 r^2} = \frac{m\upsilon^2}{r}; \quad \text{or} \quad \frac{e^2}{2\pi\varepsilon_0 r} = m\upsilon^2 \qquad (20)$$

Here, we refer to $m\upsilon^2$ in formula (20) as the centrifugal kinetic energy (Ex) of the object in circular motion

$$E_{\mathrm{X}} = m\upsilon^2 = m\upsilon r \cdot \omega = m\ \upsilon r \cdot 2\pi\nu \qquad (21)$$

where, ω is angular velocity, ν is frequency. By formulae (10) and (20), we obtain

$$\nu = \frac{2\pi^2 me^4}{h^3} = 3.28985 \times 10^{15}\,\mathrm{s}^{-1}\ ; \qquad r = \frac{2h^2\varepsilon_0}{\pi me^2} = 1.05835 \times 10^{-10}\ \mathrm{m} \qquad (22)$$

Bohr's quantization condition became

$$m\upsilon r = m\ \upsilon \cdot 2a = \frac{h}{\pi} \qquad (23)$$

By formulae (22) and (23), we obtain that the radius of the ground state hydrogen atom is twice the Bohr radius, and the frequency is completely consistent with the Rydberg frequency. At this point, we have completely solved the half frequency problem encountered by Bohr.

The de Broglie's standing wave thought is scientific basis of quantum theory

The quantization proposed by Bohr in 1913 is just a hypothesis. After ten years, de Broglie found a scientific basis for quantization, that is, a standing wave principle in physics. According to de Broglie's thought of matter wave [13], "all microscopic particles in motion are accompanied by a wave-like propagation associated with the particle. This wave-phenomenon associated with the moving particle is characterized as matter-waves which are expected to be localized in the vicinity of the particle. " Each matter wave exists in a form of a standing wave packet and propagates in the same phase with matter (particle). In physics, standing wave must have a specific-numerical wavelength. The particle is not only the center of its wave packet, but also a wave source of its matter wave. However, people's understanding deviated from an original meaning of de Broglie matter wave[13], and it is "remaining profoundly misunderstood and incomplete until the now fading end of the twentieth century"[14].

In order to correctly understand the essence of de Broglie's matter wave, we first consider a matter wave formula proposed by de Broglie [13, 14].

$$\lambda_{\mathrm{d}} = \frac{h}{p} = \frac{h}{m\upsilon} \qquad (24)$$

where λ_{d} is the wavelength of the matter wave. For a ground state hydrogen atom, its matter wave must be a standing wave. Therefore, in formula (24) both λ_{d} and h are constants. Then, the p or $m\upsilon$ must also be a constant. The λ_{d} equivalent to the wavelength of a circular oscillation wave, i.e, $\lambda_{\mathrm{d}} = 2\pi r_{\mathrm{d}}$. Here, we call r_{d} de Broglie's radius for a circular wave oscillation. That is

$$\lambda_{\mathrm{d}} = 2\pi r_{\mathrm{d}} = \frac{h}{m\upsilon} \qquad (25)$$

By formulae (8) and (25), we get

$$r_{\mathrm{d}} = \frac{h^2\varepsilon_0}{\pi me^2} = 5.3 \times 10^{-11}\ (\mathrm{m}) \qquad (26)$$

Formula (26) indicates that de Broglie's radius and Bohr's radius are the same, and the de Broglie's wavelength is the circumference of Bohr's circular orbit.

In the ground state hydrogen atom, de Broglie's wave of the electron is a standing wave. In physics, a standing wave is periodic oscillatory wave. No matter taking length or time as coordinates, a standing wave can always return to its starting point in the same waveform state after one cycle, and starts a new cycle from the starting point. This characteristic of standing wave limits that its wavelength can only be a length of specific value or constant rather than a length of any continuous change. That is, the wavelength of standing wave has a characteristics of quantization. In this way, Bohr's quantization hypothesis has theoretical basis. The wavelength, orbital radius, orbital angular momentum, and energy of electrons in atoms are all quantized.

The wavelength of a standing wave can only increase or decrease with the integral multiple of a reference wavelength. Similarly, there are quantization in orbital radius, orbital angular momentum (M) and energy (E). Namely

$$\lambda_n = n\lambda; \quad r_n = nr; \quad M_n = nM = m\upsilon r_n = \frac{nh}{\pi} \quad (n = 1,\ 2,\ 3\ \cdots) \qquad (27)$$

where λ, r and M are minimum values, and n is principle quantum number.

$$\lambda_z = \frac{1}{z}\lambda; \quad r_z = \frac{1}{z}r; \quad M_z = \frac{M}{z} = m\upsilon r_z = \frac{h}{z\pi} \quad (z = 1, 2, 3, \cdots) \qquad (28)$$

where λ, r and M are maximum values and z is nuclear charge number.

Formula (27) shows that the minimum orbital angular momentum of hydrogen atom is a constant, h/π, (of course, according to Bohr model, it is $h/2\pi$, or $\hbar$), and other values are integer multiples of this minimum value. This result fully conforms to the usual quantization rules. Formula (28) can apply to elements with z > 1.

Energy level formula of hydrogen atom

For any energy level, it can be obtained from equations (20) and (27)

$$\frac{e^2}{2\pi\varepsilon_0} = m\upsilon r_n \cdot \omega_n r_n \qquad (29)$$

Assigning v_n is the frequency of the n^{th} energy level, and $\omega_n = 2\pi v_n$. We obtain

$$\frac{e^2}{2\pi\varepsilon_0} = \frac{nh}{\pi} \cdot 2\pi v_n \cdot nr \qquad (30)$$

After simplification, we get

$$hv_n = \frac{1}{n^2}\frac{e^2}{4\pi\varepsilon_0 r} \qquad (31)$$

According to Planck's energy quantization rule and the habit of taking negative bound energy, $hv_n = -E_n$, we have

$$E_n = -\frac{1}{n^2}\frac{e^2}{4\pi\varepsilon_0 r} = -\frac{1}{n^2} \times 2.17988 \times 10^{-18}\ \text{J, or } -\frac{1}{n^2} \times 13.6057\ \text{eV} \qquad (32)$$

Note that in this derivation the radius r_n is not proportional to n^2, but ω_n or v_n is inversely proportional to n^2.

When hydrogen atom absorbs or radiates a photon, the energy of the photon is equal to the energy level difference before and after the electron transition. Namely

$$\Delta E = E_{n_2} - E_{n_1} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot \frac{e^2}{4\pi\varepsilon_0 r}, \quad (n_1 < n_2) \qquad (33)$$

Orbital shape and orbital angular momentum of each energy level for hydrogen atom

The ground state of a hydrogen atom has only one standing wave. When excited, the wavelength of this standing wave increases with an integer multiple of the radius (n), and the area of the sphere increases with the square of the integer (n^2). The number of standing waves that the sphere can accommodate also increases with the square of the integer (n^2). The number of waves accommodated on the same sphere is the number of orbits. Therefore, the ground state only has one orbit; When n =2, there are 4 orbits; When n =3, there are 9 orbits; That is, the n^{th} energy level has n^2 orbitals.

There is only one electron in hydrogen atom, and all of energy levels are degenerate states for the same n. Only the ground state orbit with n = 1 is a circular orbit, and all of other excited state orbits are formed by excitation of photon or electric field, so they are elliptical orbits[5]. Photons are transverse waves that only change the major axis of an elliptical orbit and do not change the minor axis of the ellipse. r_n is the average radius of the elliptical orbit of the n^{th} energy level.

In Figure-2, a and b represent long radius and short radius of each elliptical orbital respectively, and

$$n = \frac{a+b}{2}; \quad l = \frac{a-b}{2} \qquad (34)$$

where n is the ratio of the average radius of an ellipse with the radius of its ground state orbital, and corresponds to the principal quantum number in quantum mechanics; l is the eccentricity of the ellipse orbital and corresponds to the angular quantum number in quantum mechanics. Their orbital angular momentum is

$$M_n = n \cdot \frac{h}{\pi} = \frac{a+b}{2} \cdot \frac{h}{\pi} \qquad (35)$$

It should be pointed out again that for an elliptical orbital obtained here, the nucleus is located at the center of the ellipse. This is different from Sommerfeld's elliptical orbital. His elliptical orbit is similar to a planetary orbit in the solar system, and the atomic nucleus is located at a focus of the ellipse.

The absorption mechanism of hydrogen atoms for photons has been discussed in detail in the original text [5], and will not be repeated in this article.

Electronic transition and Emission of photon wave train

For steady-state hydrogen atom, wavelength, radius, angular momentum, energy and wave packet area are quantized. They are not continuous functions and cannot be differentiated and integrated. However, in the process of absorption or emission of hydrogen atomic spectral lines, the electron will transition

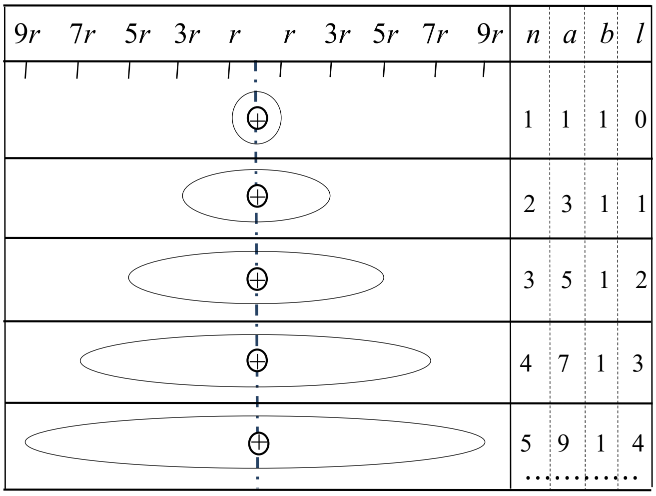

Figure 2. *Schematic diagram of orbital shapes and orbital parameters in hydrogen atom.*

between two energy levels. A transition process often takes nanosecond or even longer, and the electron needs to rotate around the nucleus millions of times or more. The better the monochromaticity of the spectral line, the longer the transition time, and the more times the electron rotates. In the process of electron transition, wavelength, radius, angular momentum, energy, wave packet area and even quantum number become continuously variable functions. Therefore, when studying the radiation mechanism and details, the continuous quantity during the transition can be differentiated and integrated.

Non-quantized changes during the transition

An electron in low-energy orbit will transition to high-energy level when it absorbs a photon. The mechanism and details of the electron absorbing radiation have been discussed in detail by the author [5]. If a transition process occurs between n_1 and n_2, then, there must be a "quantum number" that can be continuous change from n_1 to n_2. This continuously variable positive number between n_1 and n_2 is represented by g, i.e $n_1 \leq g \leq n_2$, or $n_2 \leq g \leq n_1$. Although n_1 and n_2 must be positive integers, the g does not have to be integers. All of E, r and g can change continuously between n_1 and n_2, and can be differentiable and integrable using calculus formulae.

For a transition process, the formula (32) can be rewritten as

$$E = -\frac{e^2}{g^2 4\pi\varepsilon_0 r} \qquad (n_1 \leq g \leq n_2,\ \text{or}\ n_2 \leq g \leq n_1) \qquad (36)$$

where on the right side, only g is a variable. By taking differentiation for the formula (36) , we obtain

$$dE = -d\left(\frac{e^2}{4\pi\varepsilon_0 r \cdot g^2}\right) = \frac{2e^2}{4\pi\varepsilon_0 r \cdot g^3} dg$$
$$(n_1 \leq g \leq n_2,\ \text{or}\ n_2 \leq g \leq n_1) \qquad (37)$$

If an electron transition starts from g=n_1 and ends at $g = n_2$, the photon energy absorbed is

$$\Delta E = \int_{E_1}^{E_2} dE = \int_{n_1}^{n_2} \frac{2e^2}{4\pi\varepsilon_0 r \cdot g^3} \cdot dg = \frac{-e^2}{4\pi\varepsilon_0 r g^2}\Bigg|_{g_1 = n_1}^{g_2 = n_2} = \frac{e^2}{4\pi\varepsilon_0 r} \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad (38)$$

For a transition to absorb photon, $\Delta E > 0$, and $n_2 > n_1$. For a transition to release photon, $\Delta E < 0$, and $n_2 < n_1$. The photon frequency is

$$v = \frac{|\Delta E|}{h} = \frac{e^2}{4\pi\varepsilon_0 r h} \times \left|\frac{1}{n_1^2} - \frac{1}{n_2^2}\right| = \left|\frac{1}{n_1^2} - \frac{1}{n_2^2}\right| \times 3.28985 \times 10^{15}\,\mathrm{s}^{-1} \qquad (39)$$

The ratio of the constant in formula (39) to the speed of light is the Rydberg constant.

Emission of photon wave train in hydrogen spectrum

In 1913, Bohr pointed out that "it has not been possible to observe more than 12 lines of the Balmer series in experiments with vacuum tubes, while 33 lines are observed in the spectra of some celestial bodies", …. "the diameter of the orbit of the electron in the different stationary states is proportional to τ_2. For $\tau = 12$ the diameter is equal to 1.6 x 10^{-6} cm., or equal to the mean distance between the molecules in a gas at a pressure of about 7 mm. mercury; for $\tau = 33$ the diameter is equal to 1.2 x 10^{-5} cm., corresponding to the mean distance of the molecules at a pressure of about 0.02 mm. mercury."[8] The "τ" used by Bohr is the principal quantum number n we use today. His "the mean distance" is equivalent to "mean free path" of gas molecules in his vacuum tube.

When 12 spectral lines of Balmer series are observed, it is equivalent to n_2=14; When 33 spectral lines are observed, it is equivalent to n_2=35. Bohr's observation shows that the quantum number n of the highest excited state of hydrogen atom in vacuum tube is not infinite, but limited by the mean free path in gas. Mean free path is a thermodynamic concept, which refers to the average distance traveled by molecules between two consecutive collisions in thermal motion. The lower the temperature, pressure and density of the gas, the greater the mean free path.

The mean free path not only limits the longest diameter of the elliptical orbit of the highest excited state, but also limits the time history of the emitted photon wave train. As shown in Figure-3, the hydrogen atom A in the excited state collides with particle B at point b along the x direction. If the long axis of the orbit, the orbital angular momentum and the free path of A are orthogonal, an effective collision may occur. B atoms often do not satisfy the condition of orthogonality in three directions. Therefore, in an effective collision, only A emits photon.

Effective collision is the beginning of emitting photon wave train. The collision makes the A reverse direction move along the -x axis, and the emitted photon wave train propagates along the +x direction. When A reaches point c, it collides with particle C and the emission of photon wave train ends. The distance between b and c is the free path of A. The emission of a photon wave train is completed in a free path. The head and tail of a photon wave train always differ by one wavelength.

Which low energy level does the electron go to in the transition?

In the spectral tube, the hydrogen atoms in each excited state with hydrogen molecule are in the common thermal motion equilibrium. They obey to the Maxwell-Boltzmann statistics. When the temperature, pressure and the total number of atoms remain unchanged, the atoms at each energy level have a certain probability distribution. So which low-energy level does an electron in the high-energy level want to jump to?

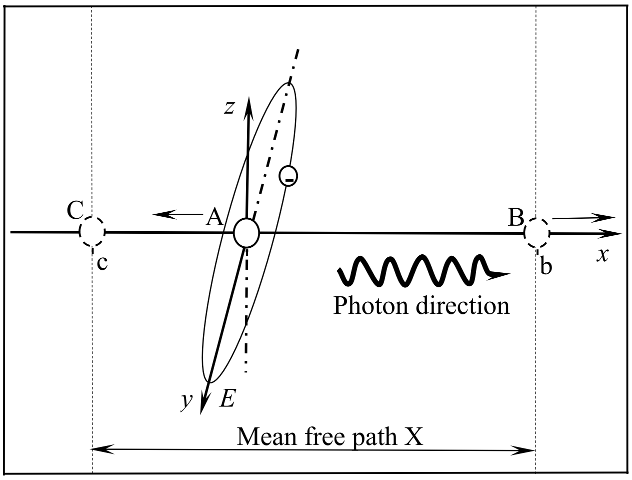

Figure 3. *Details of photon radiation. The elliptical orbit of atom A is on the xy plane, and the long axis is distributed along the y axis. The orbital angular momentum or magnetic field direction is in the z axis direction. In a free path from b to c, atom A completes the emission of a photon wave train, and the photon propagates along the x direction.*

Once the photon wave train starts emitting, its frequency has been determined. Therefore, the frequency of emitted photons depends on the effective collision at the beginning. We can determine the choice of transition by the momentum conservation of effective collision process. Under the same conditions, the higher the energy level of the excited state, the larger the collision cross section and the greater the momentum of the atom. Momentum is conserved during collision, but two atoms can exchange momentum. It can be seen from Fig.-4 that the momentum p_B of atom B is exchanged to atom A in the collision. The momentum p_A of atom A is exchanged to atom B, but atom B only receives a part of this momentum, p_1, and the another part p_2 is carried by the photon wave train. The sum of these two parts is still equal to the momentum of atom A before the collision, $p_A = p_1 + p_2$. Of course, this momentum separation is accompanied by whole emission process of the photon wave train and completed in a free path.

In thermal motion, we assume that atom A is in an excited state of $n = 5$. There are four options for its transition to low-energy orbit of $n = 4$, 3, 2 and 1, respectively. If atom B is the ground state before the collision, atom A will also become the ground state after the photon wave train is emitted, and the emitted spectral line is in the ultraviolet region. If A encounters an atom B with $n = 2$, atom A will also transition to the low energy level of $n = 2$ after momentum exchange, and the emitted spectral line is a γ line of Balmer series. Which low energy level to transition to depends entirely on the energy level state of B atom before effective collision.

Fine spectrum of hydrogen atom

Due to the existence of orbital angular momentum and spin angular momentum, each hydrogen atom has its own specific magnetic field direction. This magnetic field direction is evenly divided in the collision equilibrium of thermal motion, resulting in quantized distribution. The larger and narrower the eccentricity of the orbit, the smaller the quantized population in space. However, the larger the collision cross section, the greater the effective collision probability

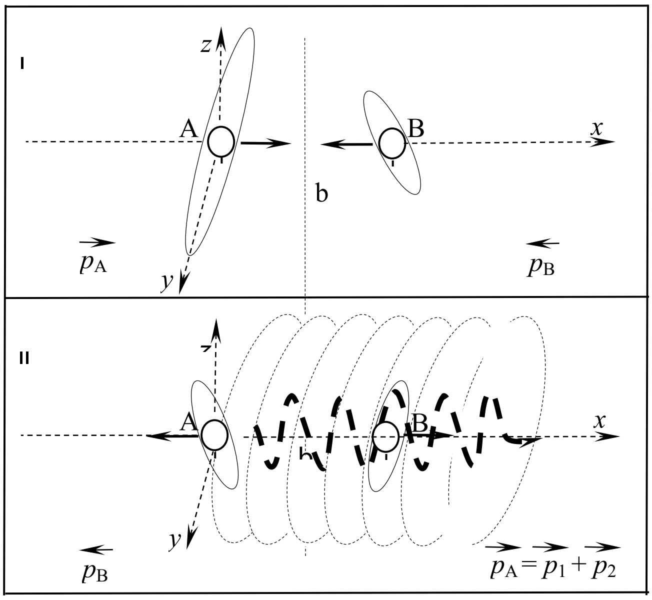

Figure 4. *Schematic diagram of photon wave train emission triggered by an effective collision. (I) Before the collision and (II) after the collision. Momentum is conserved before and after collision. By collision, the momentum of atom A is exchanged to atom B, which is divided into particle momentum and photon momentum.*

This specific atomic magnetic field orientation will affect the fine spectrum of hydrogen atoms. Therefore, the fine spectrum describes the interaction between different atoms, not the motion state of electron in an isolated hydrogen atom.

The morphology and emission mechanism of photon

The linewidth of spectral line (Δλ) refers to the difference between two adjacent wavelengths in a photon wave train. There is the following relations between the length of photon wave train (L) with the linewidth (Δλ) and wavelength (λ) [15, 16] .

$$\frac{L}{\lambda} = \frac{\lambda}{\Delta\lambda} \qquad (40)$$

Here, λ is called the eigen wavelength of the photon wave train, denote first wavelength of the head of the photon wave train as λ_h, and denote the last wavelength in the tail as λ_t. The formula (40) shows that the difference between the train head λ_h and the train tail λ_t is only one wavelength λ. Photon is released when an electron transition from high energy level to low energy level, and the higher the energy level, the longer the semi major axis of the elliptical orbit. The head of a photon wave train is related to the high energy level, and the tail is related to the low energy level. Then, there must be

$$\lambda_h = 2\lambda_t, \qquad (41)$$

Let's set $\lambda_h = \lambda + x$, $\lambda_t = \lambda - x$, by formula (41), we get

$$\lambda + x = 2(\lambda - x) \quad (42)$$

Solving Equation (42), we obtain

$$x = \frac{1}{3}\lambda, \quad \text{then,} \quad \lambda_h = \frac{4}{3}\lambda, \quad \lambda_t = \frac{2}{3}\lambda \qquad (43)$$

Taking H_α photon as an example, we explain the morphology and emission mechanism of its wave train. H_α is the first line in the Balmer spectroscopic system, which is a red spectral line emitted by an electron of a hydrogen atom when it jumps back from the third energy level (n=3) to the second energy level (n=2). Its frequency and wavelength can be calculated using equation (39)

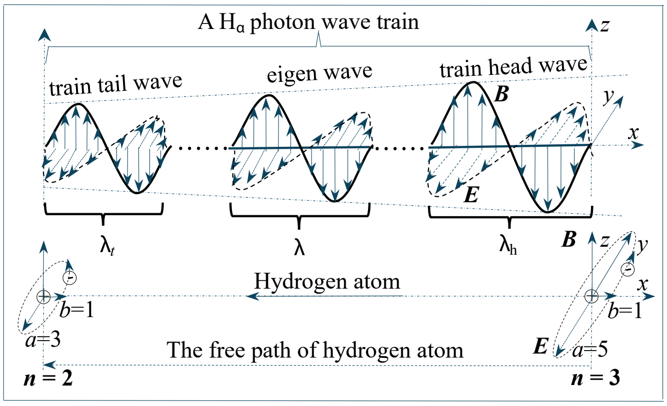

Figure 5. Schematic diagram of the morphology and emission process of an H_α photon.

$$\begin{aligned} v &= (\frac{1}{n_1^2}-\frac{1}{n_2^2})\times 3.28985\times 10^{15}\text{s}^{-1} \\ &= (\frac{1}{2^2}-\frac{1}{3^2})\times 3.28985\times 10^{15}\text{s}^{-1} \\ &= 4.56924\times 10^{14}\text{s}^{-1} \qquad (44) \end{aligned}$$

$$\lambda = \frac{c}{v} = \frac{2.997925\times 10^{8}}{4.56924\times 10^{14}} = 6.5611\times 10^{-7}\,\text{m} \qquad (45)$$

As shown in Fig.-5, when a hydrogen atom is in the excited state energy level of *n =3*, its orbital parameters *a =5* and *b=1*. In thermal motion, it collides effectively with another hydrogen atom of *n =2* energy level, exchanging momentum and releasing Hα photon wave train. First hydrogen atom moves along the - *x* direction, and the Hα photon propagates along the +*x* direction. The entire process is completed in one free path of the hydrogen atom. The wavelength λ calculated by formula (45) is eigen wavelength of Hα photon , but the wavelengths of the entire wave train vary in different segments. The wavelengths at the beginning and end of the wave train are

$$\lambda_h = \frac{4}{3}\lambda = 8.74813\times 10^{-7}\,\text{m};\ \lambda_t = \frac{2}{3}\lambda = 4.37407\times 10^{-7}\,\text{m} \qquad (46)$$

In a hydrogen spectroscopic tube, electron rotates around the nucleus tens of millions of times during one free path of a hydrogen atom in order to complete the emission of a photon wave train. Every revolution of the electron, it emits a wave of the photon wave train. Therefore, a photon wave train contains tens of millions of waves. At the end of the free path, the hydrogen atom collides with third atom (or molecule), while the emission of photon wave train is completed.

Conclusion

In the transition process of electrons, the radius, angular momentum, and energy of hydrogen atom are continuously changing functions, which can be processed by calculus, but these physical quantities are quantized at various steady states. They are all separated numerical values, not continuous functions, and are not suitable for differentiation and integration. The radius of quantization should not be replaced by a continuous radial function. Therefore, this article aiming at Rydberg's formula, adding Lorentz force to the Bohr model, accepting de Broglie's standing wave idea as the basis for quantization, and using only algebraic equations, obtains various steady-state solutions of hydrogen atoms. Thus, this processing method in this article should be referred to as the Rydberg-Bohr-de Broglie algebraic model of hydrogen atoms.

This article takes isolated hydrogen atoms as the system and only considers the electromagnetic interaction caused by orbital motion on electrons, without considering the electromagnetic interaction caused by electron spin between multiple electrons. It is not difficult to obtain a multi electron atom model based on formula (28) in the future.

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