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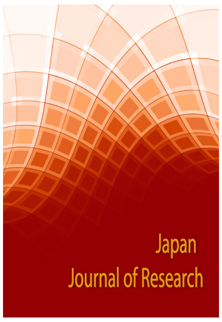

Comparison of Income Inequality Using Ratios of the Mean to the Median as a Disparity Coefficient

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Abstract

Mean and median values are widely used metrics to characterize various populations, but full attention has not been paid to the meaning of differences in these two measures. We hypothesized that the ratio of the mean (m) to the median (M) (m/M) could be useful to detect different patterns of distribution in a population. Analyzing household income of the United States, we found strong correlations between the m/M ratio and the Gini coefficient over the past 30 years (R^2: 0.9472). The Gini coefficient is predicted to be 0.37m/M. A similar tendency was observed in Japan over the same period. Furthermore, there is a strong correlation between these parameters and disposable incomes for 31 Organization for Economic Co-operation and Development (OECD) and 4 non-OECD nations. Especially for the top half of countries in the Gini coefficient, the correlation is very strong (R^2: 0.9797 and is predicted to be 0.4m/M-0.13.) Taken together, the m/M ratio may serve as an alternative index to detect social disparities when Gini coefficient is not available. The parameter may be also useful to measure deviations in distributions within populations in other fields of economics and science.

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Keywords

Inequality, Disparity, Deviation, Income, Mean over Median, Representative Value

Highlights

- The mean/median ratio may express distribution patterns in many fields of science.
- The mean/median ratio of household income correlates well with the Gini coefficient.
- The mean/median ratio may predict certain patterns of inequality independent of the Gini coefficient.

Introduction

The foundation of research in all fields of science involves quantification of observed events; measured data are expressed as fundamental statistics, usually including “representative values” with “dispersion”, also called “variability”. In scientific manuscripts, especially in papers of natural science, a mean (or average) value [1] is often used as a representative value [2]. However, if data distribution is not uniform, the mean cannot function as the representative value [3]. Because a median value is not affected by outlier events this measure can be a more reliable and representative metric in describing social phenomena. Thus, the median and mean are often reported to depict the representative value of observed events [4]. However, when both the median and the mean are simultaneously reported, it is not easy to determine what differences between the two suggest. If we calculate the mean (m) to median (M) (m/M) ratio, the meaning of the difference of the two values, namely the implication of outlier events in an observed group, should be apparent. To our knowledge, studies examining the meaning of the m/M ratio do not exist although it has long been suggested that the ratio serves as an indicator of skewedness of a distribution [5]. We sought to examine whether the m/M ratio may serve as a way to characterize a population not adequately represented by measures such as standard deviations or standard errors.

Inequalities in a society can exist in any of life’s opportunities, including education, employment, marriage, medical services or others; these inequalities can develop among locations, ages, races, sexes and other backgrounds [6]. If economic inequality underlies these other forms of inequality, a study of the gap between rich and poor should be a first step for inequality research. In

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analyzing income data, outlier values, often resulting from very rich individuals, make the mean income value a poor measure of the population. However, we hypothesized that the degree to which outlier values bias the income distribution can be effectively expressed as the m/M ratio, the ratio of mean income to median income in various societies.

The word "inequality," if used alone, means only a difference in the equality of something. As a sociological term, however, the implication is different. For instance, in the Cambridge Dictionary, "inequality" is defined as the unfair situation in society when some people have more opportunities, money, etc. than other people [7]. Although disparity itself is likely in any society, the patterns of dispersion are different between unequal and equal societies. In a theoretically equal society, the mean and the median should be identical and the dispersion should be symmetrical, whereas in societies with significant inequalities, populations would be skewed and the median would be often surpassed by the mean. If inequality in income develops, the rich become not only richer but also fewer and the poor become not only poorer but also more prevalent, elevating m and separating it farther from M.

Under the hypothesis that in a society with income inequality the m/M ratio of income mean to income median is always above one and becomes progressively greater over years, we calculated the m/M ratio of incomes in the United States (U.S.) for the past 30 years and also applied the possible use of the ratio in other societies. We also compared the m/M ratios to the Gini coefficient. For our analyses, we first tried to relate the m/M to the Gini coefficient. We then focused on the U.S. because its income statistics have less bias compared to those of developing nations and because data for the Gini coefficient are available over more years than other countries. We next analyzed Japan in a similar way. Finally, we analyzed disposable incomes of 31 OECD nations and 4 other nations.

Material and methods

Income data in the U.S. from 1953 through 2023 were obtained from the U.S. Census Bureau [8]. The Gini coefficients for household incomes were also obtained from the U.S. Census Bureau [9]. These data reflect incomes before redistribution. Because of a significant change in the data collection methodology after 1993, income data before 1993 were analyzed separately in the present study.

The Gini coefficient and household income of Japan were from annual reports published by Japanese government [10,11].

For comparison of the m/M ratio for household incomes and Gini coefficients among 35 countries (31 OECD nations except for Germany and 4 other countries), data were obtained from the OECD Income Distribution Database (IDD): Gini, poverty, income, Methods and Concepts [12]. The data were only available after redistribution as disposable incomes. Most data are from 2021 but data for 4 nations are from 2022.

Calculation of m/M and R squared of the Pearson correlation coefficient (R^2) were obtained from Excel (Version 2407) for Microsoft 365. Linear regression analysis was performed with GraphPad Prism 10.3.1.

Results

Theory and Calculations

We began this study by first examining the theoretical relationship between the Gini coefficient and m/M. The Gini coefficient, also called Gini ratio or Gini index, was developed by Corrado Gini over a century ago [13] and is now widely used to quantify income inequality [14]. The Gini coefficient is defined as a ratio of the areas on a Lorenz curve diagram, a graphical representation of wealth distribution in a population [15]. The Lorenz curve is represented by a function L(F), the cumulative portion of the total income, where F is the cumulative portion of the population. When the Lorenz curve starts at (0,0) and ends at (1,1), and where a and b are areas denoted in Figure 1, the Gini coefficient is given as:

$$G = a/(a+b) = (1/2-b)/1/2 = (1/2-\int_0^1 L(F)dF)/1/2 = 1-2\int_0^1 L(F)dF \qquad (1)$$

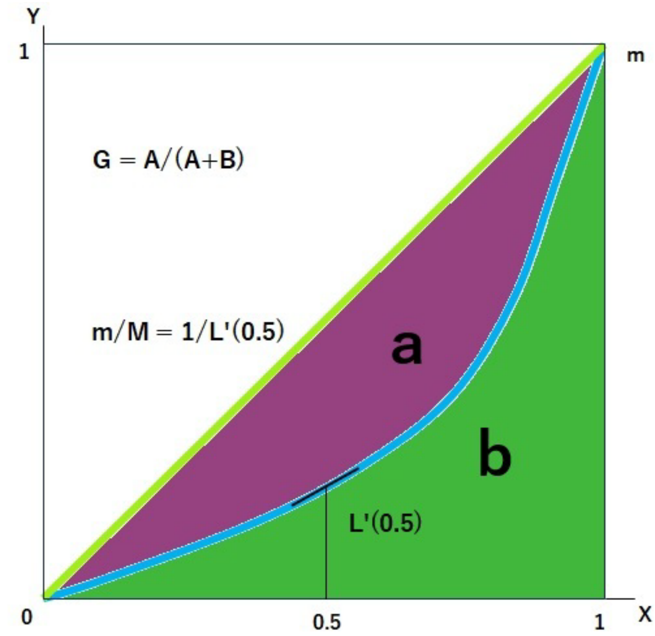


Figure 1. *m/M and the Lorenz curve. In this graph, the x-axis is denoted as the percentile of net worth ranking compared to other households. The y-axis is denoted as the cumulative amounts of net worth of households. The Lorenz curve is shown as a blue line. If the Lorenz curve is represented by a function L(F),* $\int_0^1 L(F)dF$ *represents area b. The line of equality is shown as a y = x line in light green. Where Gini coefficient is denoted as the ratio of areas a /(a+b), m/M is shown as an inverse of the slope of the Lorenz curve at x=0.5 (L'(0.5).*

And M (median) is given as:

$$M = L'(0.5) \qquad (2)$$

Where *L'(F)* is a deductive function of *L(F)*. Because m (mean) is always 1 in the Lorenz curve ending at (1,1),

$$\frac{m}{M} = \frac{1}{L'(0.5)} \qquad (3)$$

Thus, when the Lorenz curve is a concave function with a tendency, the *m/M* and Gini coefficient can be correlated. For instance, if *L(F)* is a power function, the m/M and Gini coefficient (G) are described as follows:

When $L(F) = F^n$,

$$\int_0^1 L(F)dF = [\frac{1}{n+1}F^{n+1}]_0^1 = \frac{1}{n+1} \qquad (4)$$

$$G = 1-2\int_0^1 L(F)dF = \frac{n-1}{n+1} \qquad (5)$$

$$L'(F) = nF^{n-1} \qquad (6)$$

$$L'(0.5) = n(\frac{1}{2})^{n-1} = \frac{n}{2^{n-1}} \quad (7)$$

$$m/M = \frac{1}{L'(0.5)} = \frac{2^{n-1}}{n} \quad (8)$$

$$\frac{m/M}{G} = \frac{2^{n-1}(n+1)}{n(n-1)} \quad (9)$$

Thus, if *L(F)* is a power function F^n, the *m/M* is 3-fold of the Gini coefficient when *n = 2* and when *n* is bigger than 2, *m/M / G* is less than 3, though *L(F)* is not limited to a power function.

Changes in the United States

According to the U.S. Census Bureau [8], the mean (m) and median (M) of household incomes have risen exponentially since 1953 but divergence in these values has been clear since around 1990 (Figure 2A). For instance, the median household income (M) for 2023 was $100,800 and the mean household income (m) was $135,700 resulting in an m/M ratio of 1.346. The changes in m/M ratios since 1953 are shown in Figure 2B. Accordingly, changes in the Gini coefficient (G) are shown in Figure 2C [9]. Given that the Gini coefficient (G) is 0.452 for 2023, the m/M ratio is about 3-fold higher than the Gini coefficient (m/M / G = 1.346/0.452 = 2.98). Interestingly, the increase in m/M corresponds well with the increase in the Gini coefficient (Figure 2D). Linear regression analysis revealed a very strong correlation between m/M and Gini coefficient (R^2 : 0.9836).

However, an interval is apparent in the relationship between m/M and Gini coefficient (arrow in Figure 2D), which reflects the gaps around 1992 in incomes and Gini coefficient (Figure 2B, C). We analyzed data before 1992 and those after 1993 separately (Figure 2D). Although both sets show strong correlations between the Gini coefficient and the m/M ratio (R^2: 0.8572, 0.9472 respectively), the slopes are significantly different, indicating that the relation of m/M over Gini coefficient changed around 1992. What happened during that time is discussed later

Changes in Japan

Compared to the U.S., Gini coefficient data for household incomes in Japan are not fully available but have been available every 3 years since 1990 [10,11]. Quite different from the situation in the U.S., both the mean and median of household incomes have been stagnant over these 30 years (Figure 3A). In spite of this stagnation, the ratios of m/M have been increasing (Figure 3B). More apparently, Gini coefficients have been constantly rising (Figure 3C). Although there is a positive

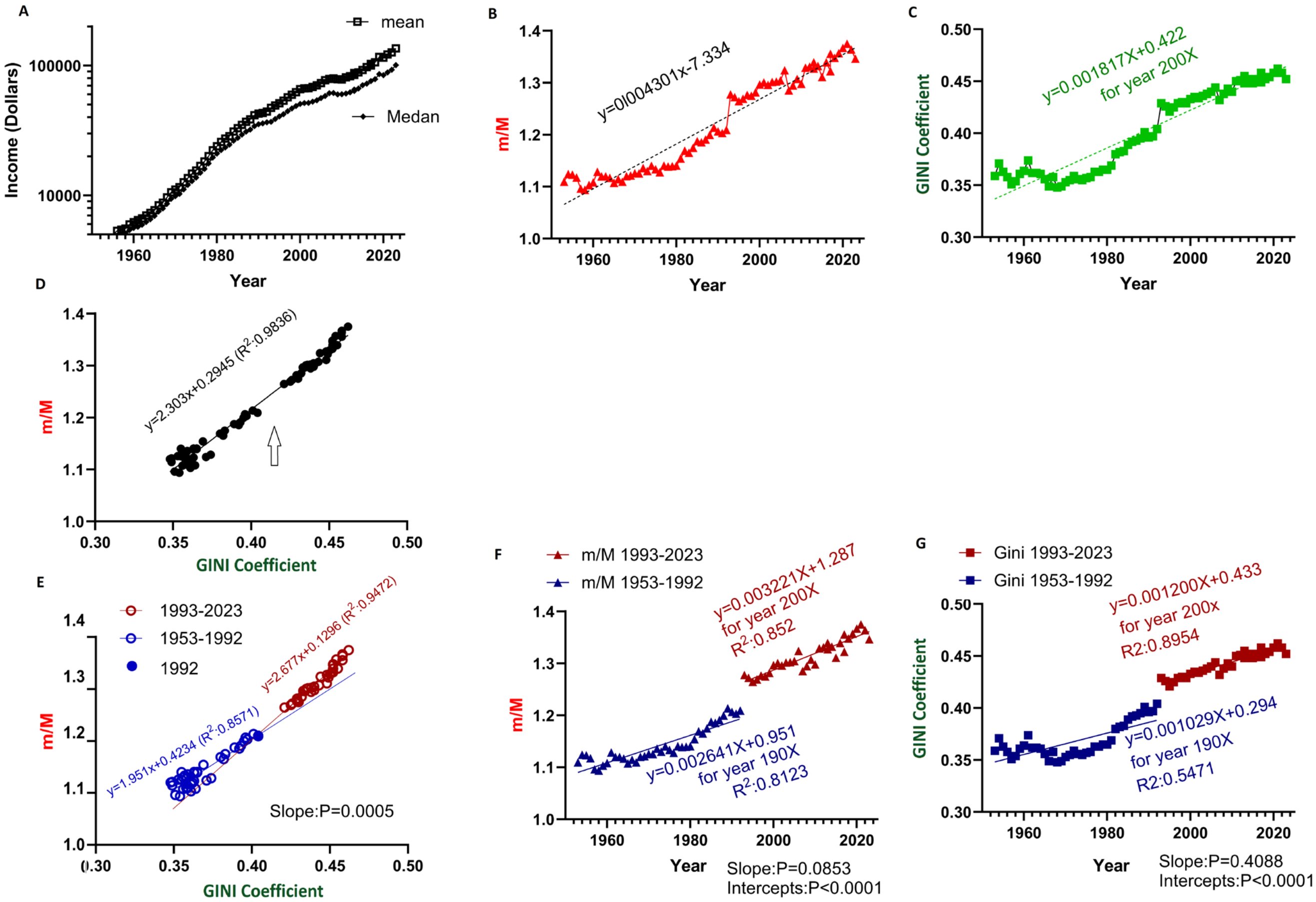


Figure 2. *Income, Gini coefficient and m/M in the United States. A. The mean and median of household income since 1953. B. Changes in the ratios of m/M since 1953. C. Changes in Gini coefficient since 1953. D. The correlation between m/M and Gini coefficient. The arrow indicates an interval that occurred between 1992 and 1993. E. Correlations were separately analyzed before 1992 (blue) and since 1993 (red). Among the blue circles the rightmost one shows 1992 (solid circle). F,G. Similarly slopes and intercepts were compared between these two groups. Slopes were not significantly altered before or after 1993 but intercepts were clearly different.*

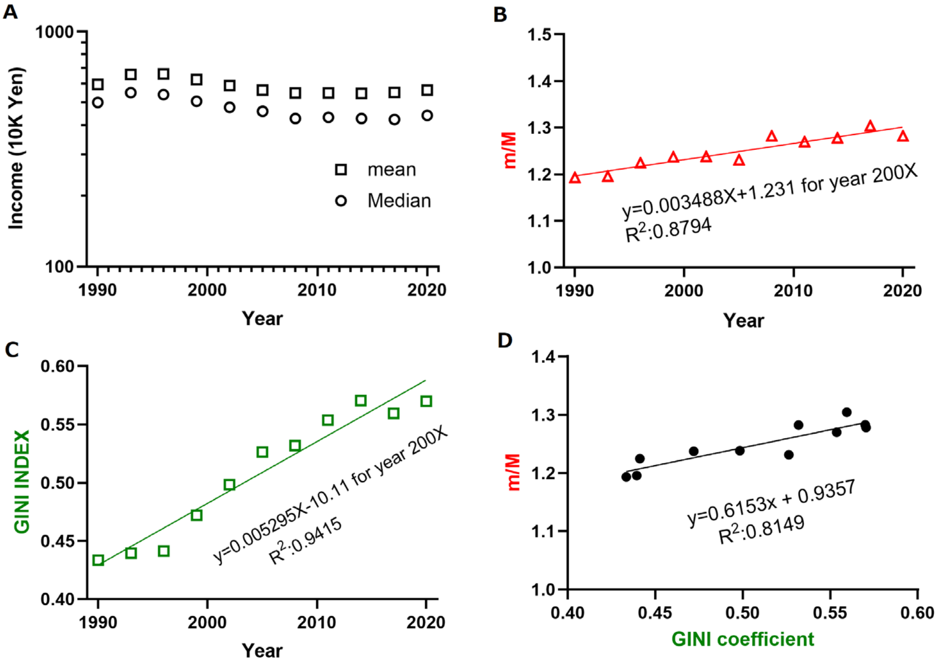


Figure 3. *Incomes, Gini coefficient and m/M in Japan. A. Although the mean and the median of household income are stagnant, a gap between mean and median has been consistently present. B. Accordingly, the ratio of m/M is gradually increasing. C. Gini coefficients reported every three years. D. The correlation between Gini coefficients and m/M ratios.*

relation between m/M and Gini coefficient (R^2: 0.8149), the slope of m/M over Gini coefficient is not as steep as that in the U.S.

Gini coefficient and m/M ratio in 31 OECD nations and 4 other nations for disposable incomes

We next determined whether there is a relationship between m/M ratios and Gini coefficients for household incomes after redistribution across multiple nations. The most recently available data provide Gini coefficients in 2021 for 33 nations and in 2022 for those 33 nations plus 2 other nations from the OECD Income Distribution Database (IDD) [12]. We calculated the m/M ratios for these 35 nations and compared these ratios to Gini coefficients (Table 1). As shown as Figure 4A, there is a close correlation between the m/M ratios and Gini coefficients (R^2: 0.9003). However, the slopes of m/M over Gini coefficients are significantly different between nations with Gini coefficients under 0.32 and those with Gini coefficients over 0.32 Gini (Figure 4B), suggesting that there are at least two relationships between Gini coefficients and m/M. It seems that the gap between m and M cannot be ignored when Gini coefficients for disposable incomes are over 0.32. Similarly, the slopes are also significantly different between nations of m/M under 1.2 and those with m/M over 1.2 (Figure 4C). Additionally, the correlation between Gini coefficients and m/M is not strong when m/M is under 1.2. The strong correlation when Gini coefficients and m/M are high might imply that the pattern of inequality is unvarying (see Discussion).

Discussion

The fundamental statistics used to describe the features of whole data sets consist of representative values and dispersion.

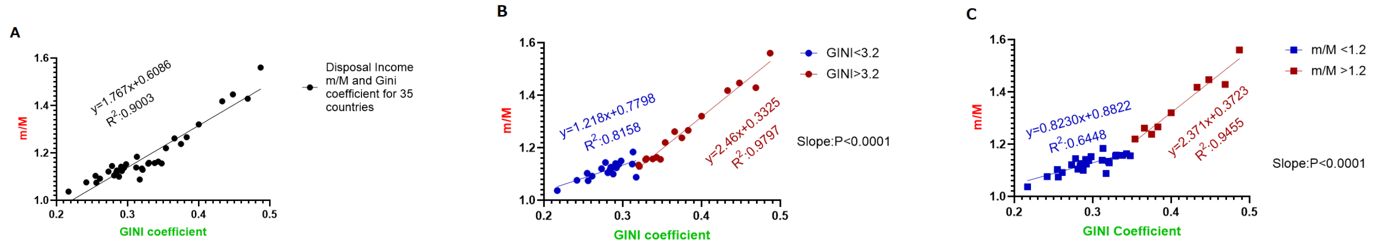


Figure 4. *The Gini coefficients and m/M ratios for disposable household incomes among 35 nations. A. Gini coefficients and the m/M ratio are highly correlated. B. The slopes of m/M over Gini coefficients are different between nations with Gini coefficients under 0.32 (blue) and those with Gini coefficients over 0.32 (red). C. The slopes of m/M over Gini coefficients are also different between nations of m/M under 1.2 (blue) and those with m/M over 1.2 (red).*

Table 1. m/M ratios for 35 nations and compared ratios to Gini coefficients

Nation	Gini coefficient	mean(m)	Median(M)	m/M	Currency
Austria	0.281	33,806	30,595	1.104952	EUR
Belgium	0.256	33,016	30,727	1.074495	EUR
Canada	0.292	58,745	52,274	1.123790	CAD
Chile	0.448	9,794,993	6,770,587	1.446698	CLP
Costa Rica	0.487	5,348,559	3,428,000	1.560256	CRC
Czechia	0.255	389,795	353,309	1.103269	CZK
Estonia	0.321	18,716	16,581	1.128762	EUR
Finland	0.273	31,949	28,500	1.121018	EUR
France	0.298	28,930	25,110	1.152131	EUR
Greece	0.312	12,269	10,780	1.138126	EUR
Hungary	0.278	3,149,725	2,751,495	1.144732	HUF
Ireland	0.291	37,718	33,040	1.141586	EUR
Israel	0.348	106,442	92,113	1.155559	ILS
Italy	0.330	23,843	20,587	1.158158	EUR
Japan	0.338	2,940,000	2,540,000	1.157480	JPY
Korea	0.329	37,060,000	32,090,000	1.154877	KRW
Latvia	0.343	13,533	11,635	1.163128	EUR
Lithuania	0.366	13,820	10,957	1.261294	EUR
Luxembourg	0.284	55,432	49,666	1.116096	EUR
Mexico	0.400	121,813	92274'	1.320123	MXN
Netherlands	0.295	34,800	30,600	1.137255	EUR
Norway	0.285	504,442	448,018	1.125941	NOK
Poland	0.261	53,120	48,650	1.091881	PLN
Portugal	0.313	14,897	12,582	1.183993	EUR
Slovak Republic	0.217	11,176	10,778	1.036927	EUR
Slovenia	0.242	19,887	18,490	1.075554	EUR
Spain	0.320	21,545	18,991	1.134485	EUR
Sweden	0.286	362,226	322,476	1.123265	SEK
Türkiye	0.433	50,802	35,830	1.417862	TRY
United Kingdom	0.354	25,197	20,656	1.219839	GBP
United States	0.375	57,680	46,600	1.237768	USD
Non-OECD economies					
Brazil	0.469	27,171	19,018	1.428699	BRL
Bulgaria	0.383	15,113	11,934	1.266382	BGN
Croatia	0.288	11,302	10,278	1.099630	EUR
Romania	0.317	34,233	31,466	1.087936	RON

Among representative values, the mean (m) is the most common statistic used in various fields of science but is a metric that is skewed by outliers [1]. Thus, for a population with skewed distributions, the median (M) can be a more reliable representative value. However, the comparison of these two representative values, m and M, is uncommon in the literature despite the fact that both are widely used. Because the mode, another representative value that is also less affected by outlier numbers, does not reflect the broader picture of a population if the population is heterogeneous [16], mode values were not used in the present study. We hypothesized that the ratio of the mean to the median (m/M) could express the appearance of disparity in a population. In this study, we calculated income distributions in the U.S. and other nations and found that the m/M ratio of household income may serve as a disparity coefficient to predict the existence of inequality.

Three types of disparity and the m/M ratio

It is likely that there are multiple patterns of income disparity [17,18] and also several patterns in the Lorenz curve. As income inequality is commonly characterized by the existence of a small number of super-rich individuals, it could also be influenced

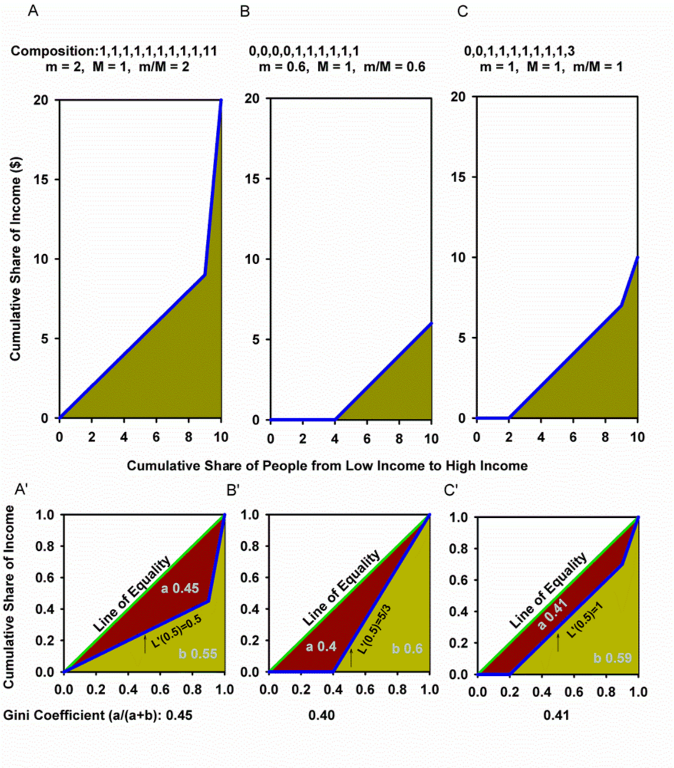


Figure 5. *Three theoretical patterns in the Lorenz curve. There are three villages A, B and C. Each total population is 10. In village A, there is one super rich individual who gains $11 while other members equally gain $1. In village B, 6 members equally gain $1 and 4 members have no income. In village C, one rich member gains $3 and 7 other members gain $1 while two other members have no income. In A', B', C, the Lorenz curves are shown in blue and the line of equality is shown in green. The Gini coefficient is double the red area, whereas m/M is the inverse of the slope of the Lorenz curve at the middle point. Although the Gini coefficients are similar among these three villages, m/M varies widely.*

by the existence of dropouts. Depending on the manner of disparity, the m/M ratio can be more than 1, less than 1, or close to 1. Because the Gini coefficient itself cannot separate patterns of disparity, the ratio of m/M could be useful to determine the type of disparity in play.

We propose three examples to explain the variation of m/M with a similar Gini coefficient (G) and the same median value (M) (Figure 5). The Gini coefficient is defined as a ratio of areas on the Lorenz curve diagram, a graphical representation of wealth distribution (Figure 5 A'-C'). Areas between the lines of perfect equality and the Lorenz curves are shown in red (a) in Figure 5, while the areas under the Lorenz curves are in brown (b); the Gini coefficient is defined as a/(a+b). As a first example, assume in a village of 10 inhabitants 9 workers earn $1 but one earns $11. In this village m/M=2; G=0.45 (See Figure 5A). The ratio m/M being greater than 1 indicates that the disparity results from the presence of a super-rich individual. In a second village of 10 inhabitants, 6 people earn $1 while the other 4 earn 0; G=0.4. The income disparity in this village results from dropouts and the m/M ratio is less than 1 (Figure 5B). In a third village, 7 earn $1, one earns $3 and the other two earn $0. In this case m/M=1 and G=0.41. In spite of the high G, m/M does not detect the presence of disparity (Figure 5C), implying that for this third type of disparity (mixed type) the m/M ratio alone is a weak index for disparity and may require additional information about the data.

While in theory there should be at least three prototypes for disparity, we found that the m/M ratio was consistently greater than 1 in the 35 countries examined and correlated well with the Gini coefficient, suggesting that the inequality in these countries reflects the first type of disparity characterized by the presence of super rich individuals. Thus, using real data the m/M ratio works well as an alternative to the Gini coefficient.

The m/M ratio of household income as an alternative disparity coefficient

We assumed that economic inequality is reflected in polarization of a minority of rich people and a majority of a poor population [19,20]. Under this assumption, mean income should consistently surpass median income. Therefore, the m/M ratio would increase as the Gini coefficient increases.

In the U.S., changes in the ratio of m/M coincide with the Gini coefficient and the m/M ratio has gradually risen over the 70-year period examined. However, with more careful inspection the slope of the rise is not necessarily constant and inequality fluctuates. This often reflects fluctuation of the mean. Since 1992 the mean rose more quickly than the median, making the m/M ratio higher. In contrast, it seems that economic recession impacts mean income more than the median. For instance, the m/M ratio fell in 2007 during the great recession. Furthermore, the ratio should also be affected by changes in taxation. A bump of the ratio in 2001 might reflect the EGTRRA (Economic Growth and Tax Relief Reconciliation Act of 2001) [21]. In 2003, changes in income taxation could affect both the rich and the middle class. In addition to the reduction of the maximal marginal tax rate, the bracket for 15% income tax was drastically increased [22]. Because of the simultaneous rise in both mean and median income, the m/M ratio remained flat.

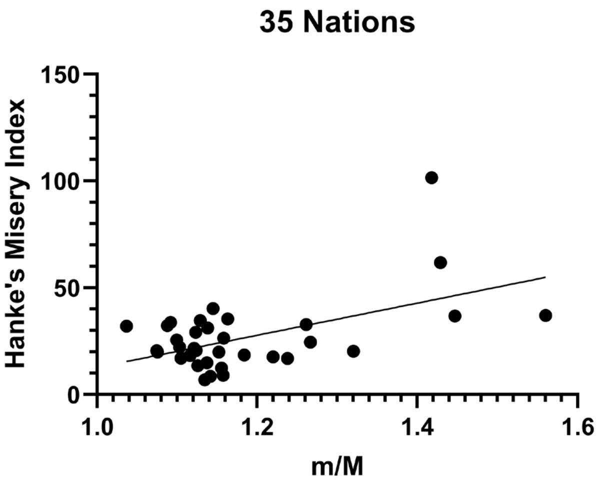
As described in the Theory/Calculation section of this paper, when the Lorenz curve is a function of a tendency, the m/M can correlate with Gini coefficient. Indeed, changes in the m/M ratio coincide with changes in the Gini coefficient (Figure 2B). Therefore, the Gini coefficient, which itself is complicated to calculate, could be roughly approximated by division of the m/M ratio of household income by 3 over these 30 years (more exactly, G=0.374m/M-0.0484). However, this formula does not apply before 1992 because the correlation between Gini coefficient and m/M before 1992 was different. As aforementioned, there are at least three types of inequity. Changes in the relation between Gini coefficient and m/M could reflect possible degeneration in the inequity. The inequity after 1992, which is characterized by high m/M, indicates existence of super rich who raise the mean. Hufe et al. have pointed out that a stable share of unfairness in inequality by the 1980s was followed by a steep rise throughout the 1990s until the present day in the U. S. [23].

The situation with household incomes in Japan differs from the U.S. Surprisingly, household incomes in both mean and median, have remained flat or declining over the 30 years examined. In this economic stagnation, the Gini coefficient has risen steadily (Figure 3C). Interestingly, the slope of m/M over Gini coefficient is more gradual, rather than steep, compared with the U. S. (Figure 2E and 3D, P<0.0001), indicating that the inequity in Japan differs from the U. S. even if the level of Gini coefficient is similarly high. The suppressed level of m/M indicates that the inequity in Japan is characterized by a majority with low income. Kitao and Yamamoto have recently reported that consumption inequality continued to rise from 1981 to 2021 [24]. Interestingly, in their report, P50/P10 (median/10th percentile and P90/P50 (90th percentile/median) have remained stable over these 20 years, implying that the composition of the majority from the bottom 10% to the top 10% has not drastically changed. Although it is not easy to simply compare their findings with ours because of differences in the raw data, it should be noted that inequity in disposable incomes may behave differently from that in incomes before redistribution.

Among 35 nations, Gini coefficients and m/M of disposable incomes after redistribution show strong correlation when these parameters are higher than certain levels. However, the correlation is weak when these parameters are lower than certain levels. Among 16 EU nations, m/M of all nations except Lithuania are less than 1.1 though Gini coefficients vary. Slovakia and Belgium show the lowest m/M and also the lowest Gini coefficients among EU nations, in spite of the fact that the means in Europe show about 3-fold differences. This observation implies that the mean alone or the median alone could not predict inequality.

The Gini coefficient, whose calculation requires large data sets to delineate a Lorenz curve, is often unavailable for small societies such as counties and cities. Even for nations, Gini coefficients are not necessarily available every year. We propose that the m/M ratio is simple and easy to calculate and can be useful to measure the degree of disparity in a population.

In addition to the Gini coefficient, the misery index, proposed by Arthur Okun in the 1960's [25], might have some relationship to m/M of incomes, since the misery index, a sum of unemployment rate and inflation rate, could reflect the degree of poverty. Among 35 nations, we compared m/M of disposable incomes to Hanke's misery index, a version of the misery index that was available in 2022 [26]. As shown in the supplemental figure, however, there is no strong correlation between m/M and Hanke's misery index (R^2: 0.2736), though nations with extremely high misery indices also show high m/M. For instance, the misery index and m/M for Brazil are 61 and 1.43, respectively; the misery index and m/M for Turkey are 101 and 1.42, respectively.



Supplementary Figure. *The relation between m/M and Hanke's misery index among 35 nations in 2022. There is no strong correlation between m/M and Hanke's misery index (R^2: 0.2736). Turkey and Brazil have the highest misery index whereas Costa Rica and Chile have highest m/M.*

Beyond income inequality

Although m/M of household income is greater than 1 in all nations examined, it could be less than 1 in theory. Furthermore, m/M must be always be less than 1 in a population or measurement that has an upper limit. For instance, the median life expectancy always surpasses the mean. In future studies, it will be interesting to examine differences in the m/M ratio for life expectancy and other variables among nations.

Conclusion

Both the mean (m) and median (M) are representative values of a population, but little attention has been paid to how the ratio of these measures can provide important information about a population. Using income data, we show that the m/M ratio is useful for predicting certain patterns of inequality and correlates well with the Gini coefficient across 35 nations. Furthermore, this ratio could be a useful indicator to express the distribution in various populations in many fields of social and natural science.

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