

Intensity of Visual Illusions Calculated from Geometric Parameters of the Stimulus

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Abstract

The angle subtended by the consciously perceived image – the active angle – corresponds to a certain area on the retina. Hypothetically, this section on the retina is displayed on an “internal visual memory screen” that is always the same size. Several of the geometrical-optical illusions are attributed to a changing active angle adapted to the extent of the stimulus. An example is given, and an algebraic expression is derived from a model. In this way, the intensity of the illusion can be traced back to the dimensions of the geometric elements of the stimulus. The visual sense is compared to a data communication channel. Its limited capacity may occasionally lead to a reduction in the active angle, enabling better resolution. This can make a celestial body appear larger. It occurs automatically close to the horizon. Other reasons are given that can reduce the active angle. Depictions of celestial bodies in paintings indicate that the active angle can vary greatly from case to case.

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General

Although it is known that the intensity of geometric optical illusions is strongly dependent on the shape, the size and the arrangement of the elements of the stimulus, there have been few attempts to put these relationships into a quantitative mathematical context [1]. The object available to the sense of sight is the retinal image. The visual system is often compared with a camera. While a camera can use different focal lengths to capture different sized sections of the environment and map them onto a specific image format of constant size, the visual system – so the underlying model here – processes differently sized sections of the image on the retina into a perception and projects the perceived impression onto an internal visual memory screen that is always the same size. This enables the visual system to perform the same task as a telephoto lens.

The zoom lens model [2,3] proposes that the attended region can be adjusted in size and predicts a tradeoff between its size and resolution because of limited processing capacities. There are geometrical-optical illusions where a length or a diameter serves as the target. Decisive – so the assumption – for the intensity of the illusion is the angle that the entire perceived image takes in relation to the angle subtended by the target. It can also be said that the angle subtended by an object – the visual angle (NN 1) [4] – is not always perceived to be the same size. The moon illusion is one example.

Although one can notice an object within the field of view (NN 2) [5] which spans up

to 210 degrees of arc (NN 3) [6], this covers too large an area to be remembered in detail. However, what we call perception in the proper sense, the impression that we consciously perceive and remember, comes just from a fraction of the field of view. Since the visual system increases or decreases this angle via top-down mechanisms [7,8], the term “active visual angle” or, shortly, „active angle“ [9] is used here. The criteria to do so are derived from the overall stimulus [7].

In geometrical-optical illusions, there is the question of which criteria are used to select a certain active angle. The aim here is to establish a model which tells the dependence of the perceived size of the target on the size of geometric parameters of the stimulus in the form of an algebraic expression.

Since the visual system can be understood as a data processing channel, the question also arises as to what influence a limited channel capacity has on the active angle. Previous studies have provided evidence for the hypothesis that transient attention can enhance spatial resolution [10]. Even depictions on paintings and drawings indicate that the active angle can vary greatly from case to case [11].

The Active Angle of Human Vision

Under average conditions in everyday life this active angle corresponds to the angle captured by a so-called standard camera lens. It is the angle of view of a camera with a focal length of 50 mm and an image area of 24 by 36 mm. Its angle is 47 degrees arc over the diagonal. (Figure 1, NN 4 [12]). For details see Eye (NN 5) [13].

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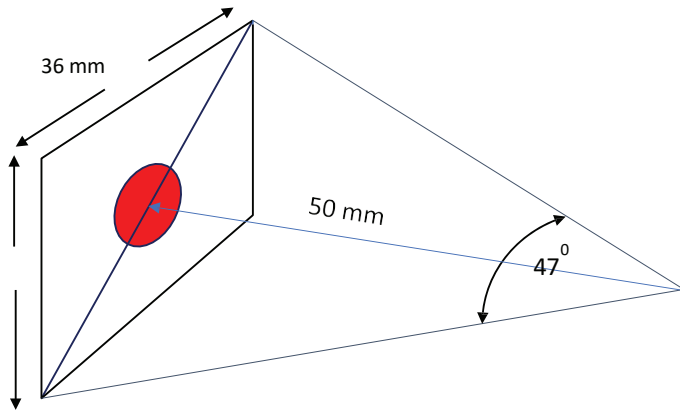


Figure 1. Angle of view of a camera with a normal – or standard – lens. This corresponds approximately to the active angle of view of the human eye. The diagonal measures 47 degrees in arc.

Since the focal length of the eye is largely fixed, the visual system can zoom in or zoom out only by selecting a smaller or larger area on the retina for further processing of the visual information. This may confirm our experience that the moon, the sun or even constellations can appear quite different sizes. In addition, it is known that, in geometrical-optical illusions, the length of a line [14], the distance between two objects [15] or the diameter of a circle, like in the Ebbinghaus [16] or the Delbeuf illusion [17,18] can be perceived quite differently, depending on the extension of the overall stimulus [19].

The Model

The current active angle from which we draw the consciously perceived image corresponds to a certain area on the retina (Figure 2). This consciously perceived image from the section chosen finally becomes visible to us on an Internal visual memory screen“which is always the same size (Figure 2, blue frames at the right side), comparable to a movie screen which would not change size when the scenery proceeds from a view over a wide landscape to a butterfly on a grass leaf. The image is also saved in this format.

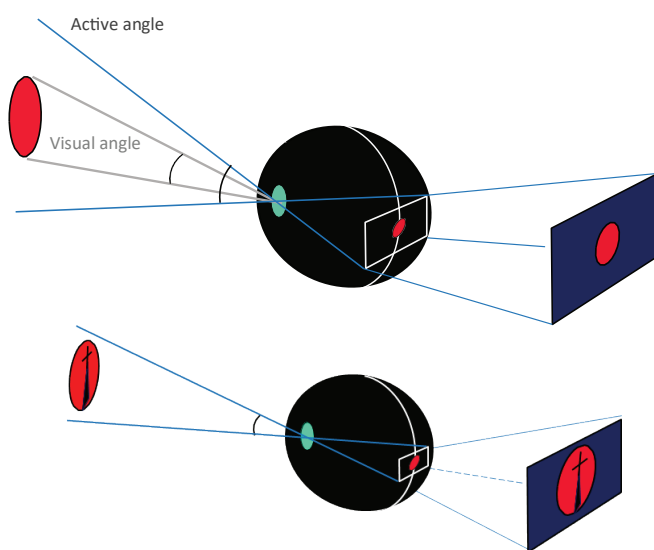


Figure 2. A larger (top) or smaller active angle of view corresponds to a larger or smaller area on the retina. If this is projected onto an „Internal visual memory screen“ of always the same size (dark blue rectangles on the right), then, with a smaller active angle, a certain part of this image will appear enlarged (bottom).

If a smaller area on the retina has been selected, corresponding to a smaller active angle, then an object of always the same size will fill a higher proportion of this angle and will appear correspondingly larger (Figure 2, bottom).

There are several conceivable causes that may trigger an increase or decrease in the active angle. Among them there are:

- The extension of the stimulus in mind.
- Limited channel capacity – the number of bits that can be processed per unit time – may produce either a highly resolved image from a smaller area or a lower resolved one from larger an area, respectively.
- The height of the observed object above the horizon [20].
- The particular interest in an object.

Size Constancy

In 1925 Erna Schur published a thesis on this topic [21]. It refers to the observation that, with increasing distance, an object does not seem to become smaller as quickly as one would expect due to the size of the retinal image. In an experiment the participants determined the apparent size of a bright disc in absolute darkness, vertically as well as horizontally, by comparison with another disc of constant size at constant distance. There were no indications of spatial depth. It was found that, in horizontal direction, at four times the distance (16 meters compared to 4 meters) the circle did not seem to have a quarter of its diameter but more than half, the exact value being 0.54 (Figure 3). This corresponds to a subjective magnification of the target or a shrinking of the active angle by a factor or 2.16. This result can also be expressed in algebraic terms (EQ 1) [22].

While the retinal image y becomes smaller in proportion to the distance of observation, x ,

$$y_{\text{(retinal)}} \sim x^{-1},$$

the perceived image shrinks less rapidly, due to

$$y_{\text{perceived}} \sim x^{-(n-1)}, \quad \text{EQ 1}$$

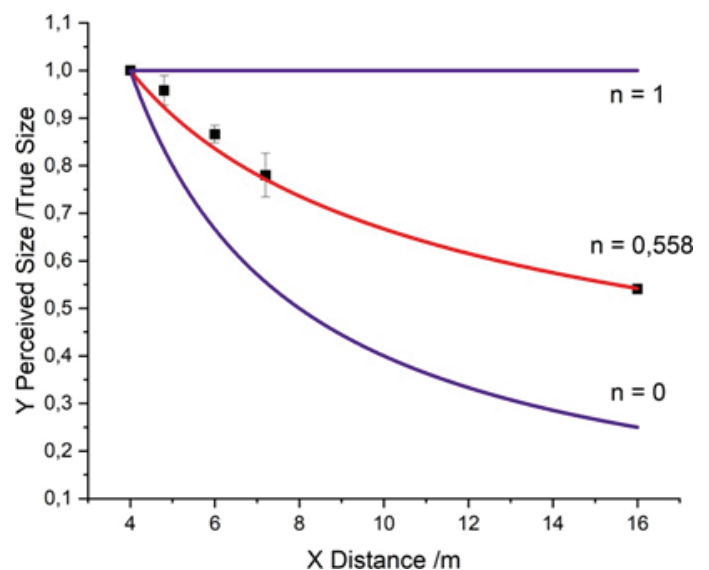


Figure 3. Bottom curve: Size of the retinal image as a function of distance. Red curve: Perceived size as a function of distance. n means the size constancy parameter. The straight horizontal line corresponds to $n = 1$; in this case the apparent size would be independent of distance.

with n being the size constancy parameter (≈ 0.5 for horizontal and ≈ 0.3 for vertical direction of observation). The result $n \approx 0.3$ for vertical direction of observation means that raised objects appear smaller. From this, other authors drew the conclusion that the further away the horizon is, the larger the moon must appear. Erna Schur did not share this opinion, in fact she contradicted it.

The question arises as to whether the term “size constancy” refers exclusively to size perception of a target at different distances or on two objects of different sizes at the same distance as well. In other words, whether a small isolated retinal image automatically triggers a smaller active angle. The example given in the next chapter is based on this idea.

The Mirrored Triangles' Illusion

An example of the stimulus is given in Figure 4, with the distance of the dips serving as the target. In Figure 5, bottom, pairs of triangles are shown in a mirror image arrangement, their dips being 60 units apart. Their shape varies from elongated in horizontal direction to almost equal-sided to narrow and tall. The area of one triangle is always 756 square units. In an experiment, the perceived distance between the triangles was determined as a function of their shape.

The Algebraic Function: The goal was to find an algebraic expression giving the apparent size as a function of the horizontal as well as the vertical extension of the stimulus.

In a first attempt [23] a function had been fitted where the apparent distance depended on one variable only. In contrast, two variables are used here, and an extended data set as well.

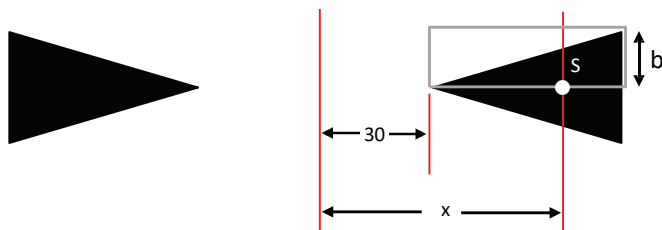


Figure 4. Example of the stimulus. The distance between the tips (60 units) serves as the target. x (the distance of the center of gravity S from the origin) serves as the independent variable. The variable b corresponds to half of the basis of one triangle. The size of one triangle (corresponding to the dashed rectangle) measures 756 square units.

From the dashed rectangle in Fig 4 one obtains $b = \frac{756}{(x-30)(\frac{3}{2})} = \frac{504}{(x-30)}$ units. The algebraic expression for the apparent distance $y_{\text{perceived}}$ of the tips to be fitted to the data is

$$y_{\text{perceived}} = (m \cdot x + n \cdot b)^{(k-1)} = (m \cdot x + n \cdot 504/(x-30))^{(k-1)} \quad \text{EQ 2}$$

With m , n and k as the parameters to be determined. If k were

zero this would mean that in case the active angle decreases in proportion to the stimulus while the target stays constant, the apparent separation of the dips would increase in inverse proportion to this. The parameter k indicates how far the measured variable (the apparent size of the target) deviates from this assumption.

Procedure: 10 volunteers took part in the experiment. 15 different transparencies were displayed on a screen, one after the other in random order. First, the stimulus was shown for 4 seconds. Then a stack of 7 horizontal lines of different lengths was added for another 4 seconds, followed by a break of 4 seconds. Participants noted the number of the standard that appeared to be closest to the target.

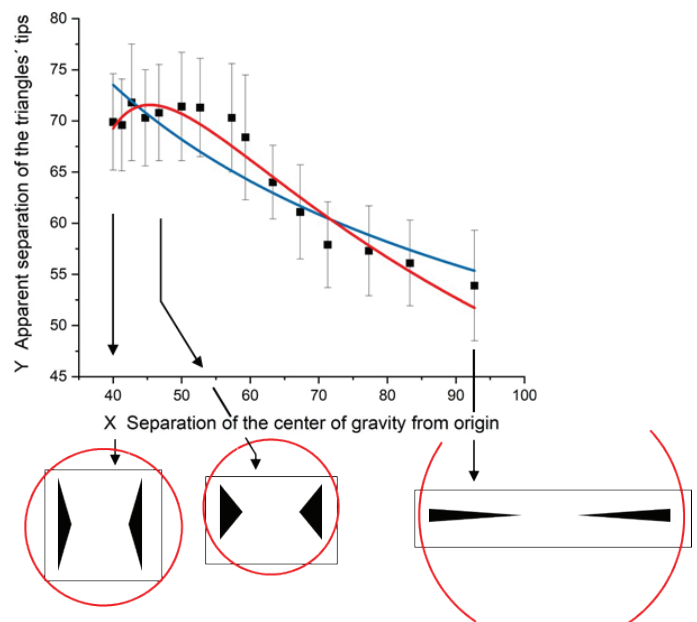


Figure 5. Perceived length of the target plotted over the separation of center of gravity from the origin. The red curve gives the fit of EQ 2. To fit the blue curve, the vertical extension b of the triangles has not been taken into account. Below there are examples of stimuli. The circles indicate their extension, which in turn determines the size of the active angle.

Result: The red curve in Figure 5 gives the result of fitting EQ 2. The apparent distance $y_{\text{perceived}}$ depends on the variable x (the horizontal extension) as well as on the vertical extension b of the triangles. In this way the intensity of the illusion can be traced back quantitatively to the geometric shape of the stimulus.

To give an example, while the horizontal extent of the stimulus ($\sim x$) is reduced by a factor of 2 (from 90 to 45 units), the apparent size of the target increases by 33%. This means that the active angle has become smaller by the same amount. The blue curve is obtained when the vertical extension of the triangles is not taken into account. Table 1 gives the parameters.

Table 1. Result of the fitting procedure

	$y_{\text{perceived}}$	m	n	k
EQ 1	$m \cdot (x)^{(k-1)}$	256(44)		0.662(43)
EQ 2	$(m \cdot x + n \cdot 504/(x-30))^{(k-1)}$	3.74(250) E-5	1.75(1.36) E-5	0.298(81)

This shows that it is primarily the horizontal extension that determines the intensity of the illusion. However, the influence of the increasing vertical extension b is sufficiently strong to bend the curve downwards towards smaller x , so a maximum of the perceived distance occurs if the triangles are fairly narrow and high. The red circles in Figure 5 indicate the relative extensions of the stimuli, which in turn determine the size of the active angle. Figure 6 shows two different examples in comparison.

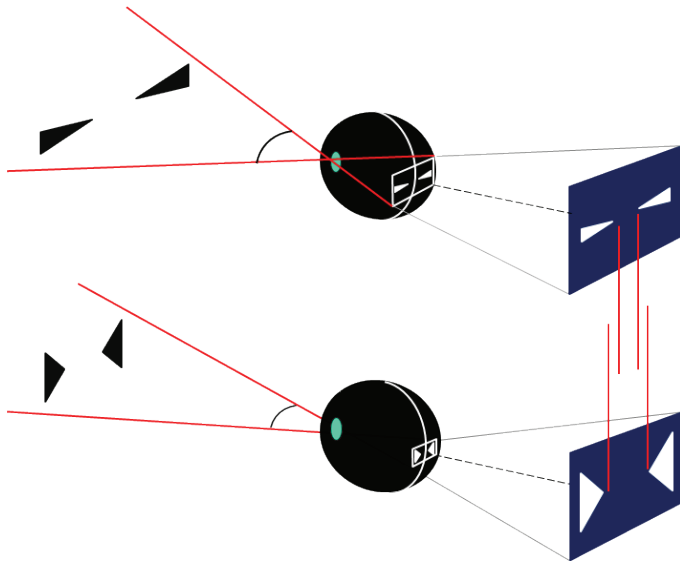


Figure 6. Increase of the apparent size of the target due to adaption of the active angle to the size of the stimulus. A smaller active angle makes the distance appear larger (bottom)

The Moon Illusion

There is a wealth of literature on the observation that the moon appears significantly larger on the horizon than high in the sky [24]. The visual system can be regarded as a data transmitting and processing channel. While in optical transmission the image is broken down into pixels, in visual perception the information is digitized by the ganglion cells and then reduced in the course of further processing [25].

There is only a limited number of picture elements to be processed per second. This allows either a larger area with low resolution or a small area with high resolution to be displayed and perceived. There are several possible reasons for reducing the active angle and thus increasing the resolution and make a celestial body appear larger: One is good visibility in a clear atmosphere or a sharply drawn silhouette in front of the celestial body (Figure 2, bottom). However, the most important factor seems to be the perceptibility of the horizon [20]. During the course of evolution, it may have been advantageous to be able to recognize details preferably on the horizon as well and as early as possible. This also explains why the raised moon appears so small.

In the eye, more information per unit area can be obtained from the retina in the immediate vicinity of the fovea due to the lower number of photoreceptor cells connected to one ganglion cell [26].

The subjective impression of the size of celestial bodies is often depicted in paintings [11].

Data Availability

The author declares that the data supporting the findings are available within the paper.

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