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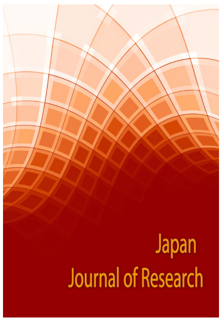

Battery Energy-Stokes' Relationships Between The Gap of Two Zeta Zeros in the BHTM

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Abstract

According to my previous paper [3] about the charge ratio of this writer's proposed Gabriel Trumpet black hole Toy model, one may consider the black hole as a charged battery with the computed boundary charge ratio -- i/j for a special designed (with those Riemann Zeta Zeros height) Taylor series like inverted cone approximation to the Trumpet toy model. In the present paper, this writer will continue the computation procedure to calculate the charge density together with the stored energy etc at the boundary. In addition, this writer also discovers a conversion relationships (by matrix operations) linking the parametrized matrix to a given surface ω (curl and divergence) with the metric tensor (1st, 2nd, 3rd, 4th fundamental forms) for the investigated object's surface with the enclosed curve c. Besides, this writer also uses the software Maple Soft programming scripts as well as the traditional human type calculation to verify or justify the equivalence between the Stokes' Theorem and the Green's Theorem and also the Divergence Theorem etc.

Actually, the Green's theorem and the Divergence Theorem [1] are most likely to be used for the 2-Dimensional surface areas and 3-Dimensional volumes while the Stokes' Theorem is applied in the higher dimensional cases. In fact, we may still have high dimensional curl and divergence with respect to their definitions. At the same time, the Green and the Divergence Theorems can be well interpreted or connected to the famous Maxwell's equations in physics or electrodynamics.

Last but not least, there are some philosophical implications such as the cognitive feelings or insights behind the Stokes' Theorem rather than those line integral, surface area and volume relationships for our physics.

Introduction

In my previous paper [3], this writer has suggested that we may use the U.S.A. mathematical software such as the Matlab to generate a piece-wised surfaced Gabriel Trumpet Black Hole Toy Model which may be associated to the practical quantization of the actual black hole or even the loop quantum gravity. From my previous paper [3], this writer also suggests that we may apply the complex contour integral to compute the boundary or even the charge that stayed between the gap of the two layered zeta zeros, say ξi and ξj. In the following content, this writer will try to calculate the capacitance together with the energy stored (and also the up-thrusting force) that links with such gap between the aforementioned two layered zeta zeros.

Energy – Volume Relationships

Current I = ∂Q/∂t

Energy = ∂E = I^2 R ∂t

or Power = ∂E/∂t = $(\partial Q/\partial t)^2$ R

As the inverted cones is used as an approximation to the Trumpet Black Hole Model [], hence, we may have the charge ratio as:

$$\frac{\Delta chargeQfortherectangle\xi_i}{\Delta chargeQfortherectangle\xi_j} = \frac{i}{j} \text{ or } i = jk$$

where k1 is a constant

Thus, the corresponding power ratio is:

$$= \frac{P_{\xi i}}{P_{\xi j}} = \left[\frac{(\Delta Q_i)^2}{(\Delta Q_j)^2}\right] R$$

$$= \left[\frac{(i)^2}{(j)^2}\right] R$$

$$= \left(\frac{i}{j}\right)^2 R$$

$$= k_1^2 R$$

But at the same time, according to Schrödinger Equation, we may have []:

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$$\partial E = \left(\frac{\partial Q}{\partial t}\right)^2 R\partial t$$

$$\text{Power} = \frac{\partial E}{\partial t} = \frac{\Delta\left(\frac{\hbar^2 k^2}{2m}\right)}{\Delta t}$$

Substitute back into the aforementioned energy ratio equation, we may then have:

$$\left(\frac{P_{\xi i}}{P_{\xi j}}\right) = \frac{\Delta m_{\xi j}}{\Delta m_{\xi i}} = k_1{}^2 R$$

or

$$m_{\xi j} = m_{\xi i} k_1{}^2 R$$

$$\left(P_{\xi i}\right) = P_{\xi j} k_1{}^2 R$$

Also, the volume ratio is in fact:

$$\frac{V_i}{V_j} = \frac{\frac{1}{3}\pi r_i^2 h_i}{\frac{1}{3}\pi r_j^2 h_j} = \frac{r_i^2 h_i}{r_j^2 h_j}$$

In fact, when the boundary of the inverted cone is just considered as a surface area with a line curved, then by the Stokes' Theorem, the volume integral is equal to the surface area, i.e. $\int(\nabla\cdot F)dv = \oint_c F\cdot dA$ or the Gauss Divergence Theorem.

Actually, the Generalized Stokes' Theorem is [5]:

Let ω be a smooth (n – 1) form with compact support on an oriented, n-dimensional manifold with boundary Ω, where ∂Ω is given the induced orientation. Then $\int_\Omega d\omega = \int_{\partial\Omega}\omega$. But as the Generalized Stokes' Theorem will finally be reduced back to the Gauss Divergence Theorem or even the Divergence Theorem, hence, in my proposed study case, we may have:

$$\frac{\frac{1}{3}\pi r_i^2 h_i}{\frac{1}{3}\pi r_j^2 h_j} = \frac{\pi r_i^2}{\pi r_j^2}$$

Or

$$h_i = h_j$$

(N.B. When the boundary of the surface area is considered to be a bounded space,

then we may apply the Divergence Theorem as follow:

$$\iiint(\nabla\cdot F)dV = \oiint_s F\cdot dS$$

This writer will NOT directly prove the equivalence between the divergence theorem and the Stokes' theorem but I will verify or justify the relationship between these two theorems through the following hemisphere case study together with some the Maple Soft's programming scripts.

A Case Study of the Hemisphere with Maple

Let's first consider M, the upper hemisphere of the unit sphere [2] $S^2 \subset {}^3$ as a sub-manifold with boundary. That is:

$$M = \left\{x \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1, x_3 \geq 0\right\}$$

The boundary is the circle in the (x1, x2)-plane:

$$\partial M = .\left\{x \in \mathbb{R}^3 : x_1^2 + x_2^2 = 1, x_3 = 0\right\}$$

The vector field flux or curl(F) around the hemisphere is:

$$F = y^3\underline{i} - x^3\underline{j} + x\underline{k}$$

Then the volume element on the manifold dA is given by:

dA(v, w) = |(vxw)| where v, w are vectors in TpM [16].

Moreover v x w is aligned with n = n^x **i** + n^y **j** + n^z **k**

Next, we may consider the parametrization of M [2] by using the spherical coordinates as below:

$$\varphi(\theta,\psi) = \left(\cos(\theta)\sin(\psi), \sin(\theta)\sin(\psi), \cos(\psi)\right)$$

for U given by π<θ< π, 0<ψ≤ π/2 where ψ = π/2 corresponds to the boundary points.

Then by considering the 2-form, we may get [1]:

$$\begin{aligned}\omega &= F^x dy\wedge dz + F^y dz\wedge dx + F^z dx\wedge dy\\ &= y^3 dy\wedge dz + \left(-x^3\right)dz\wedge dx + x dx\wedge dy\\ &= y^3\underline{n}^x dA + \left(-x^3\right)\underline{n}^y dA + x\underline{n}^z dA\end{aligned}$$

as the wedge product $dx_i \wedge dx_j$ is only the area or the determinant of the enclosed surface A with the orientation vector $\underline{\mathbf{n}}^k$ [1].

Obviously, $\int_{\partial M}\omega = \int_{\partial M}\left(y^3, -x^3, x\right)\cdot\left(n^x, n^y, n^z\right)dA$

$$= \int_{\partial M}\left(y^3 n^x dA + -x^3 n^y dA + x n^z dA\right)$$

Hence, one may compute:

$$d\ \omega = 3y^2\frac{\partial y}{\partial x}dx\wedge dy\wedge dz + \left(-3x^2\right)dy\wedge dx\wedge dz + \frac{\partial x}{\partial z}dz\wedge dy\wedge dx$$

$$= \left(3y^2\frac{\partial y}{\partial x} - 3x^2\frac{\partial x}{\partial y} + 1\frac{\partial x}{\partial z}\right)dx\wedge dy\wedge dz$$

$$= \nabla\cdot F dx\wedge dy\wedge dz$$

$$= div\ F\ dV$$

Therefore, for the above case study, we have actually verified or justified that:

$$\int_M \nabla\cdot F dV = \int_M d\omega = \int_{\partial M}\omega = \int_{\partial M} F\cdot n dA$$

Similarity, if we go ahead for a step in the Stokes' Theorem in the case study of the hemisphere and assume the validity of the Green's theorem such that: $\int_s \nabla\times F\cdot \mathrm{d}S = \int F\cdot \mathrm{d}r$ we may still verify or justify the Stokes' Theorem for the surface area and the line integral, but this time, the writer will employ the software MapleSoft to work for such Stokes-Green Theorem verification purpose, i.e.

$$\int_M \nabla\times F\cdot \mathrm{d}M = \int_M d\omega = \int_c \omega = \oint_c F\cdot \mathrm{d}r$$

First, let us consider the following relations with the proposed parametrization:

$$\varphi(\theta,\psi) = \left(\cos(\theta)\sin(\psi), \sin(\theta)\sin(\psi), \cos(\psi)\right)[2]$$

Then by Maple Soft, we have:

Step 1: with(DifferentialGeometry); with(Tools);

Step 2: DGsetup('[f = dgform(0), dx = dgform(1), dy = dgform(1), dz = dgform(1), sigma = dgform(2)]', [1], M)

Output: frame name: M

Step 3: with(VectorCalculus); [15]

Output: [`&x`, `*`, `+`, `-`, `.`, `<,>`, `<|>`, About, AddCoordinates, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, VectorCalculus[Nabla], Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series]

Step 4: F := <y3, -x3, x>

Output: F:= (y3)ex + (-x3)ey + xez

Step 5: Dginfo("AbstractForms")

Output: [f, dx, dy, dz, σ]

Step 6: ω_x := evalDG(dy &wedge dz)

Output: ω_x :=dy ∧ dz

Step 7: ω_y :=evalDG(dx &wedge dz)

Output: ω_y :=dx ∧ dz

Step 8: ω_z :=evalDG(dx &wedge dy)

Output: ω_z :=dx ∧ dy

Step 9: ω := evalDG(-x3*&ω_y + y3*&ω_x + x*ω_z)

Output: ω:= -x3 dx ∧ dz + y3 dy ∧ dz + x dx ∧ dy

Next, the parametrized ω is thus:

$$\left[\begin{array}{l} P(\varphi(\theta,\psi))\left|\left(\begin{pmatrix}\frac{\partial\varphi_2}{\partial\theta} & \frac{\partial\varphi_3}{\partial\theta}\\ \frac{\partial\varphi_2}{\partial\psi} & \frac{\partial\varphi_3}{\partial\psi}\end{pmatrix}\right)\right| + Q(\varphi(\theta,\psi))\left|\left(\begin{pmatrix}\frac{\partial\varphi_3}{\partial\theta} & \frac{\partial\varphi_1}{\partial\theta}\\ \frac{\partial\varphi_3}{\partial\psi} & \frac{\partial\varphi_1}{\partial\psi}\end{pmatrix}\right)\right| \\ +Q(\varphi(\theta,\psi))\left|\left(\begin{pmatrix}\frac{\partial\varphi_1}{\partial\theta} & \frac{\partial\varphi_2}{\partial\theta}\\ \frac{\partial\varphi_1}{\partial\psi} & \frac{\partial\varphi_2}{\partial\psi}\end{pmatrix}\right)\right| \end{array}\right] d\theta d\psi$$

$$= \begin{pmatrix} P(\varphi(\theta,\psi)) & Q(\varphi(\theta,\psi)) & R(\varphi(\theta,\psi)) \\ \frac{\partial\varphi_1}{\partial\theta} & \frac{\partial\varphi_2}{\partial\theta} & \frac{\partial\varphi_3}{\partial\theta} \\ \frac{\partial\varphi_1}{\partial\psi} & \frac{\partial\varphi_2}{\partial\psi} & \frac{\partial\varphi_3}{\partial\psi} \end{pmatrix} \left|\left(\begin{pmatrix} 1 \\ \partial\theta \\ \partial\psi \end{pmatrix}\right)\right| \quad [2]$$

which is just a Curl (<F^x, F^y, F^z>) [1] for the parametrization of the vector field dot the product to the differential of the associated parametrized curve r(t) = (t, θ(t), ψ(t)).

(N.B. There may be a continuous recursive substitution of the parametric equation such that we may finally get:

$$\omega = |parametic\text{matrix}A||parametic\text{matrix}B||parametic\text{matrix}N|\left|\left(\begin{pmatrix} 1 \\ \partial\theta \\ \partial\psi \end{pmatrix}\right)\right|$$

(N.B. The corresponding pull-back (or in a similar way of the push-forward) complex differential form may be computed as below by the Maple Soft:

Let the multi-dimensional vector-coordinates be (x, y) where *x* denotes the real number axis, y denotes the imaginary axis or *yI* and $z = x + yi$, $\bar{z} = x - yi$ (which may be defined as the vector addition or subtraction between the real and imaginary vector ordered pair (*x, y*) [4]. Then

$$dz = dx + i\,dy \quad and \quad d\bar{z} = dx - i\,dy$$

where $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n)$, [7]

$z = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$ & $\bar{z} = (x_1 - y_1, x_2 - y_2, \ldots, x_n - y_n)$.

We also define the homomorphism $T_v \to \mathbb{C}$ is given by:

$d\,z_j : \frac{\partial}{\partial x_j} \mapsto 1$ and $d\,z_j : \frac{\partial}{\partial y_j} \mapsto 1$ where $i = \sqrt{-1}$ and d zj sends the other basis elements $\frac{\partial}{\partial x_k}, \frac{\partial}{\partial y_k}, k \neq j$ to 0.

Let $C \subset \mathbb{R}^2$ [1] be a simple closed curve, $D \subset \mathbb{R}^2$ where the region consisting of *C* and its interior. Let $\alpha: D \to \mathbb{C}^3$ be the parametric surface

$$\alpha\,(u, v) = (u, v, 1 - u - v), \text{ or } (u, v, 1 - u + v)$$

(N.B. u -v is also true but the computation is the same, this writer will not repeat. In fact, the real ℝ is just a subset of the ℂ, and ℂ also contains ℝ, so my proposed mathematical computation framework is still valid in such sense.)

with S = α (D) [1] and ∂S is the boundary curve of S. Let η be the one-form

$$\eta = -z\,d\,x - 2x\,d\,y + 4y\,d\,z$$

then by Stokes' theorem [1], we have:

$$\int_{\partial\alpha} \eta = \int_{\alpha} d\eta$$

with the orientation induced by the parametrization which in fact may be viewed as a complex differential form (by using Maple Soft with suitable parametric transformation, say *α (u, v) = (u, v, 1-u-v)* [1]; the addition/subtraction of the real & imaginary part of the complex number in practice and this writer wants to note that the present coordinate system itself is a 3-dimensional vector space;) to the associated complex manifold or even in the topic of complex algebraic geometry. In fact, the real manifold is a sub-manifold of the complex or in other words it is contained in the complex manifold. Hence, my proposed mathematical theory should be developed in a correct way. Furthermore, for the parametrization of a surface to a boundary curve ∂S of a curve S such that:

$$\eta = -z\,d\,x - 2x\,d\,y + 4y\,d\,z$$

then by the Stoke Theorem,

$$\int_{\partial\alpha} \eta = \int_{\alpha} d\eta$$

As $d\eta = -dz \wedge dx - 2dx \wedge dy + 4ydy \wedge dz$

we may calculate its pullback,

$$\alpha^* d\eta = -(-du - dv) \wedge du - 2du \wedge dv + 4dv \wedge (-du \wedge -dv) = du \wedge dv \quad [1]$$

To evaluate the above Stoke's theorem integral on the right-hand-side, we may thus calculate the exterior derivative by the Maple Soft: [1]

with(DifferentialGeometry);

with(LieAlgebras);

DGsetup([x, y, z], M)

η:= evalDG(-z d x – 2x d y + 4y d z) [4] output: η:= -z d x – 2x d y + 4y d z

η1:=ExteriorDerivative(η)

output: η1:= -2 dx ∧ dy + dx ∧ dz + 4 dy ∧ dz

DGsetup([u, v], M2)s

output: frame name: M2

theata := Transformation(M2, M, [x = u, y = v, z = 1 - u – v]) [11]

output: theata := [x=u, y=v, z=1- u -v]

Pullback(theata, η1)

output: du ∧ dv

(N.B. We may still obviously deduce a recursive parametrize pullback such that:

$$\gamma^*,\sigma^*(d(\beta^*(d(\alpha^*(d\eta)))) = |r_u Xr_v| \gamma^*, \ldots, \sigma^*(d(\beta^*(d(du \wedge dv)))$$
$$= |r_u Xr_v||r_k Xr_l|, \ldots, \gamma^*(d, \ldots, (dk \wedge dl))$$
$$= |r_u Xr_v||r_k Xr_l|, \ldots, |r_m Xr_n| dm \wedge dn$$

In general, we may prove the above result by the mathematical induction method.)

DGsetup([x, y], M2):

DGsetup([x, y, z], E3):

with(DifferentialGeometry); with(LieAlgebras);

DGsetup([x, y], E2); DGsetup([x, y, z], E3);

S := Transformation(E2, E3, [x = x, y = y, z = 1 - x – y]) [6]

output: S := [x=x, y=y, z=1- x- y]

Pushforward(S, [D_x, D_y]) [6]

output: [D_x-D_y, D_y-D_z]

J := Tools:-DGinfo(S, "JacobianMatrix")

output: J:= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{pmatrix}$

with(MTM):

J1 := transpose(J)

output: $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$

S1 := Transformation(E2, E3, [x = x, y = y, z = z]) [10]

output: S1:= [x=x, y=y, z=z]

C:= J1(Vector) ˙["D_x" ,"D_y" ,"D_z"] [9]

output: C:= $\begin{pmatrix} \text{D\_x-D\_z} \\ \text{D\_y-D\_z} \end{pmatrix}$

which is just giving the same result as the push-forward and hence we have verified that the push-forward is the same as the transformation. By the way, the first two columns of the Jacobian matrix forms the bases for the transformation which are two linearly independent vectors.

Certainly, we may prove the above proposed result by the induction method.)

On the other hand, we may have the push forward from the Maple Soft:

Therefore, the equation is reduced to

$$\int_M \nabla \times F \cdot \mathrm{d}M = \int_M .Curl(< F^x, F^y, F^z >). \begin{pmatrix} \partial(r_1(t)) \\ \partial(\theta(t)) \\ \partial(\psi(t)) \end{pmatrix} .d\,M$$

where r1(t) = t.

In fact, $\int_c \omega = \int_c \left(sin^4 t - cos^4 t\right) dt$

On the other hand, if we parametrize the vector field by another r2(t) = (cost(t), sin(t)), then:

$$\oint_c F \cdot \mathrm{d}r = \oint_c < y^3, -x^3, x > .d\left(cos(t), sin(t)\right)$$

By applying Maple Soft programming scripts for $\int_c \omega = \oint_c F \cdot \mathrm{d}r$ we may actually have:

Step 10: r := <cos(t), sin(t)> [12]

Output: r:= (cos(t))ex + (sin(t))ey

Step 11: dr := diff(r, t)

Output: dr:= (-sin(t))ex + (cos(t))ey

Step 12: ans := DotProduct(F, dr)

Output: ans := -sin(t)*y3 – cos(t)*x3

Step 13: ans := subs([x = cos(t), y = sin(t)], ans)

Output: ans := -sin4(t) – cos4(t)

Step 14: $w := \int_0^{\pi} ansdt$

$$= -\frac{3\pi}{4}$$

Therefore, $\oint_c F \cdot \mathrm{d}r = \oint_c < y^3, -x^3, x > .d\left(cos(t), sin(t)\right) = \frac{3\pi}{4}$ in the anti-positive direction.

Alternatively, let's end the justification or verification with the Curl integral below:

$$\int_M \nabla \times F \cdot \mathrm{d}M = \int_M \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3 & -x^3 & x \end{pmatrix} \cdot \begin{pmatrix} idy \wedge dz \\ jdx \wedge dz \\ kdx \wedge dy \end{pmatrix}$$

d x = <2x, 0, 0>, d y = <0, 2y, 0>, d z = <0, 0, 2z>

dx∧dy = 4xy; dy∧dz = 4yz; dx∧dz = 4xz

By Maple Soft, we have:

Step 15: with(linalg):

curl(F, [x, y, z])

Output: $< 0 , -1 , (-3x^2-3y^2) >$

With the substitution (cos(t), sin(t), 1) and computation, we finally get [8]:

$$\int_M \nabla \times F \cdot \mathrm{d}M = \int_0^{\pi} . < 0, -1, \left(-3cos^2(t) - 3sin^2(t)\right) > \begin{pmatrix} 4[sin(t)](0) \\ 4[cos(t)]0 \\ 4sin(t)cos(t) \end{pmatrix} \cdot \begin{pmatrix} -sin(t) \\ cos(t) \\ 1 \end{pmatrix} \cdot dt$$
$$= \int_0^{\pi} . \left(-sin^4 t - cos^4 t\right) dt$$
$$= -\frac{3\pi}{4}$$

As by the Step 14,

$\oint_c F \cdot \mathrm{d}r = \oint_c < y^3, -x^3, x > .d\left(cos(t), sin(t)\right) = \frac{3\pi}{4}$ in the anti-positive direction.

Therefore, we have verified that $\int_M \nabla\times F\cdot \mathrm{d}M = \oint_c F\cdot \mathrm{d}r$ [14]

$$\int_{\partial M} d\omega = 3y^2\frac{\partial y}{\partial x}dx\wedge dz + \left(-3x^2\right)\frac{\partial x}{\partial y}dy\wedge dz + \frac{\partial x}{\partial z}dz\wedge dy$$

$$= 3y^2\left(4x\right)\left(z\right)\frac{\partial y}{\partial x}3x^2\left(4y\right)\left(z\right)\frac{\partial x}{\partial y} + \left(4x\right)\left(y\right)$$

Hence, by the parametrization (sin(t), cos(t), 0), we may get:

$$\int_{\partial M} d\omega = \int_0^\pi \cdot\left(-\sin^4 t - \cos^4 t\right) dt$$

$$= -\frac{3\pi}{4}$$

Obviously, $\int_{\partial M}\omega = \int_{\partial M}\left(y^3,-x^3,x\right)\cdot\left(n^x,n^y,n^z\right)dA$

$$= \int_{\partial M}\left(y^3 n^x dA + -x^3 n^y dA + x n^z dA\right)$$

By the substitution r(t) = (cos (t), sin (t), 0), we may get:

$$\int_{\partial M}\omega = \int_{\partial M}\left(\sin^3(t)n^x dA + -\cos^3(t)n^y dA\right)$$

$$= \int_0^\pi\left(\sin^3(t)n^x + -\cos^3(t)n^y\right)dA$$

$$= \int_0^\pi -\sin^4(t) - \cos^4(t)dA$$

$$= -\frac{3\pi}{4}$$

Hence, from the above case study of Green-Stokes' Theorem, we may verify or justify that:

$$\int_M \nabla\times F\cdot \mathrm{d}M = \int_M d\omega = \int_c \omega = \oint_c F\cdot \mathrm{d}r$$

(N.B. Also at the boundary, according to the Green's theorem (for 2D objects while Stokes' Theorem is for 3D objects), the ratio of the surface area is also equal to the line integral [1], i.e.

$$\int_S \nabla\times F\cdot \mathrm{d}S = \oint_c F\cdot \mathrm{d}r$$

$$\frac{\pi r_i^2}{\pi r_j^2} = \frac{2\pi r_i}{2\pi r_j} = \left(\frac{2i\pi}{2j\pi}\right)$$

$$\frac{q_{\xi i}}{q_{\xi j}} = \frac{r_i}{r_j} = \left(\frac{i}{j}\right)$$

(Use the inverted cone as an approximation to my proposed Trumpet Black Hole model)

$\frac{V_i}{V_j} = \frac{i\left|\sqrt{\Im(\xi_i)^2}\right|}{j\left|\sqrt{\Im(\xi_j)^2}\right|}$ as $\frac{r_i^2}{r_j^2} = \left(\frac{i}{j}\right)$ at the boundary [3] or even

$$\frac{r_i}{r_j} = \pm\sqrt{\frac{i}{j}},$$

$$\frac{q_{\xi i}}{q_{\xi j}} = \frac{i}{j}$$

$$= \frac{\left|\Im(\xi_j)\right|V_i}{\left|\Im(\xi_i)\right|V_j}$$

When $h_i = h_j = |I(\xi_i)| = |I(\xi_j)| = |I(\xi_n)|$ (where $|I(\xi_n)|$ is the last item of the proposed inverted approximated cone) that is also at the boundary, these factors will cancel each other and thus we have:

$$\frac{q_{\xi i}}{q_{\xi j}} = \frac{i}{j} = \frac{V_i}{V_j}$$

(N.B. For $\frac{r_i}{r_j} = \pm\sqrt{\frac{i}{j}}$, the surface integral or line integral energy ratio is

$$\frac{q_{\xi i}}{q_{\xi j}} = \frac{r_i}{r_j} = \pm\sqrt{\frac{i}{j}},$$

$\frac{P_{\xi i}}{P_{\xi j}} = \pm\sqrt{\frac{\Delta m_{\xi j}}{\Delta m_{\xi i}}} = \pm k_1\sqrt{R}$ Therefore, it is in theory mathematically possible that there may be the negative energy which implies the state has less energy than the empty space, but an in-depth discussion is out of the present paper's scope.)

Where else, in the normal case other than the boundary of the inverted cone,

$$\rho_i = \frac{Q_i}{V_i}$$

$$= \frac{i}{i\sqrt{\left|\Im(\xi_i)\right|^2}}$$

$$= \frac{1}{\sqrt{\left|\Im(\xi_i)\right|^2}}$$

$$= \frac{1}{\left|\Im(\xi_i)\right|}$$

$$\frac{\rho_i}{\rho_j} = \frac{\left|\Im(\xi_j)\right|}{\left|\Im(\xi_i)\right|}$$

$$\frac{P_{\xi i}}{V_{\xi j}} = \frac{m_i}{i}$$

Or the Ratio of the Power Density is:

$$\frac{Pd_{\xi i}}{Pd_{\xi j}} = \frac{jm_{\xi i}}{im_{\xi j}}$$

When $h_i = h_j = |I(\xi_i)| = |I(\xi_j)| = |I(\xi_n)|$ (where $|I(\xi_n)|$ is the last item of my proposed inverted approximated cone) that is also at the boundary condition, then we may have the ratio of power density per volume as:

$$\frac{Pd_{\xi i}}{Pd_{\xi j}} = \frac{jm_{\xi i}}{im_{\xi j}}$$

Now, by considering the voltage potential difference:

$$\Delta v = -\oint_{\xi_i}^{\xi_j}\frac{1}{2\pi l\varepsilon_0}\frac{Q}{zeta(z)}dz$$

$$= -\frac{Q(i-j)I}{l\varepsilon_0}$$

But Q is just the difference between the zeta roots' plate ξ_i and ξ_j or $j(k_1-1)$, thus we may have:

Hence, the capacitance C is:

$$\Delta C = \frac{\Delta Q}{\Delta V}$$

$$= \frac{\frac{i-j}{l\varepsilon_0}}{Q(j-i)I}$$

$$= -\frac{QI\left(i-j\right)^2}{l\varepsilon_0}$$

The energy stored between two layers of the Riemann Zeta non-trivial zeros, say ξ_i and ξ_j is therefore [4]:

$$W = \frac{1}{2}\Delta C(\Delta v)^2$$

$$= \frac{1}{2}\left[\frac{\frac{i-j}{l\varepsilon_0}}{Q\left(j-i\right)I}\right]\left[\frac{Q\left(j-i\right)I}{l\varepsilon_0}\right]$$

$$= \frac{1}{2}\left[-\frac{Q\left(i-j\right)^2 I}{\left(l\varepsilon_0\right)}\right]$$

$$= \frac{1}{2} - \frac{j\left(k_1-1\right)\left(i-j\right)^2 I}{\left(l\varepsilon_0\right)}$$

Certainly, if we were given the mirror imaged inverse energy value or the power ratio from the observations or experiments etc, we may still have the chance to estimate back the layered gaps of the Riemann Zeta zeros.

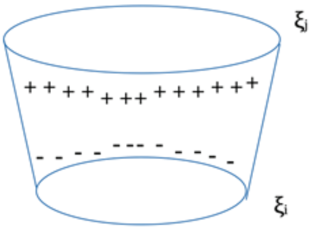

Figure 1. *A 3D-sectional view of the accumulated charge between the gap of the two layered Zeta-Zeros, ξ_i & ξ_j of my Black Hole Toy Model.*

(N.B. Actually, for the Green's Theorem, we have: $\iint_D\left(Q_x - P_y\right)dA = \oint Pdx + Qdy$ where $dA = dxdy$. When $Q_x - P_y = k$, the left hand-side is reduced into $\iint_D kdA$ which is in practice an area with a magnification factor of k. If k = 1, we have the area ratio:

$$\frac{2n_i r_i \pi I}{2n_j r_j \pi I} = \frac{\oint_{\xi_i}\frac{1}{Zeta\left(z\right)}dz}{\oint_{\xi_j}\frac{1}{Zeta\left(z\right)}dz} = \frac{\pi r_i^2}{\pi r_j^2}$$

But in practice, the area of the trapezoid as shown in figure 1 is just:

$$\frac{\left(2\pi r_j + 2\pi r_{j-1}\right)\left(\Im\left(\xi_j\right) - \Im\left(\xi_{j-1}\right)\right)}{2} = \pi\left(r_j + r_{j-1}\right)\left(\Im\left(\xi_j\right) - \Im\left(\xi_{j-1}\right)\right)$$

The summation of all these area A is just:

$A = \lim_{N\to\infty}\pi\left(r_N\Im\left(\xi_N\right) - r_1\Im\left(\xi_1\right)\right)$ as the result of the telescopic cancellation.)

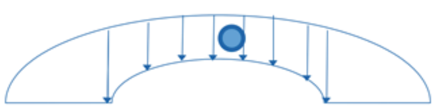

Figure 2. *A cross-sectional view of a charged particle located at the gap between two layered Zeta Zeros, ξi & ξj, of my Black Hole Toy Model, an induced force will be produced that is perpendicular to this figure on the paper sheet. Such kind of design may be applied in the electric propellant rocket engine. In addition, we may also modify the design of the blade-less fan with such kind of electric field for an additional and greater electrical induced up-thrusted force or a greater of the air flowing out etc. In fact, the key for an improvement of the efficiency of the fan is in the rate of change of curvature around its circular area or surface. i.e. How we may design the fan's surface curvature such that it will achieve its maximum. Certainly, the afore-mentioned electrical force lifting thrust design may also be used for the Magnetic Levitation train etc.)*

(N.B. Another interesting thing is that we may treat the black hole's inner structure just as what this author described before as several layers of capacitors under the condition(s) that if an enough potential has been built up and finally released their kinetic energy in terms of intense heat and X-ray radiation. Then we may observe the internal structure of the black hole. Therefore, if we can set up a similar electrical and electronic appliance where an animal body – for testing only, is charged just like the above case black hole internal layered capacitor(s), until the X-ray (& intense heat) are emitted, then we may depict some clear X-ray photo of the internal structure of the testing animal(s).)

Discussion: Maxwell's Equations and Super-conducting

With reference to the parametrization for ω, we may have [2]:

$$\left[P\left(\varphi\left(\theta,\psi\right)\right)\left|\left(\begin{pmatrix}\frac{\partial\varphi_2}{\partial\theta} & \frac{\partial\varphi_3}{\partial\theta}\\ \frac{\partial\varphi_2}{\partial\psi} & \frac{\partial\varphi_3}{\partial\psi}\end{pmatrix}\right)\right| + Q\left(\varphi\left(\theta,\psi\right)\right)\left|\left(\begin{pmatrix}\frac{\partial\varphi_3}{\partial\theta} & \frac{\partial\varphi_1}{\partial\theta}\\ \frac{\partial\varphi_3}{\partial\psi} & \frac{\partial\varphi_1}{\partial\psi}\end{pmatrix}\right)\right| + Q\left(\varphi\left(\theta,\psi\right)\right)\left|\left(\begin{pmatrix}\frac{\partial\varphi_1}{\partial\theta} & \frac{\partial\varphi_2}{\partial\theta}\\ \frac{\partial\varphi_1}{\partial\psi} & \frac{\partial\varphi_2}{\partial\psi}\end{pmatrix}\right)\right|\right]d\theta\, d\psi$$

$$= \begin{pmatrix}P\left(\varphi\left(\theta,\psi\right)\right) & Q\left(\varphi\left(\theta,\psi\right)\right) & R\left(\varphi\left(\theta,\psi\right)\right)\\ \frac{\partial\varphi_1}{\partial\theta} & \frac{\partial\varphi_2}{\partial\theta} & \frac{\partial\varphi_3}{\partial\theta}\\ \frac{\partial\varphi_1}{\partial\psi} & \frac{\partial\varphi_2}{\partial\psi} & \frac{\partial\varphi_3}{\partial\psi}\end{pmatrix}\left|\left(\begin{pmatrix}1\\ \partial\theta\\ \partial\psi\end{pmatrix}\right)\right|$$

as $\begin{pmatrix}\frac{\partial\varphi_i}{\partial\theta} & \frac{\partial\varphi_j}{\partial\theta}\\ \frac{\partial\varphi_i}{\partial\psi} & \frac{\partial\varphi_j}{\partial\psi}\end{pmatrix}$ may be transformed into the modern notation of the metric tensor $\begin{pmatrix}E & F\\ F & G\end{pmatrix}$ by the matrix operation, hence the aforementioned parametrized curl of the vector field ω also may be expressed in terms of the metric tensor $\begin{pmatrix}E & F\\ F & G\end{pmatrix}$ so as the mirror inverse of the metric tensor $\begin{pmatrix}E & F\\ F & G\end{pmatrix}$ in terms of $\begin{pmatrix}\frac{\partial\varphi_i}{\partial\theta} & \frac{\partial\varphi_j}{\partial\theta}\\ \frac{\partial\varphi_i}{\partial\psi} & \frac{\partial\varphi_j}{\partial\psi}\end{pmatrix}$. Or we may compute the (1st, 2nd, 3rd, 4th) fundamental form for the differential geometry [] in terms of the curl to ω with the parametrization $\begin{pmatrix}\frac{\partial\varphi_i}{\partial\theta} & \frac{\partial\varphi_j}{\partial\theta}\\ \frac{\partial\varphi_i}{\partial\psi} & \frac{\partial\varphi_j}{\partial\psi}\end{pmatrix}$ O or the vector field and flux etc. Hence, we may investigate those properties like the arc length, area and the curvature, of the expected boundary surface. In its mirror inverse way, the $\begin{pmatrix}\frac{\partial\varphi_i}{\partial\theta} & \frac{\partial\varphi_j}{\partial\theta}\\ \frac{\partial\varphi_i}{\partial\psi} & \frac{\partial\varphi_j}{\partial\psi}\end{pmatrix}$ may be expressed in terms of the metric tensor $\begin{pmatrix}E & F\\ F & G\end{pmatrix}$. That is we may compute the divergence, curl, line integral, surface area and the volume relationships for the vector field or flux in terms of the metric tensor for the expected boundary surface of the enclosed curve c. Alternatively, we may apply the multi-dimensional Taylor series expansion for the parameterized surface function and proceed for the corresponding Stoke's theorem computation.

In brief, this writer has just established a linking between the arc length, area and the curvature etc properties to the divergence, curl of the line integral, surface area and the volume at the boundary surface of the enclosed curve c for the vector field or flux through the matrix conversion between the metric tensor $\begin{pmatrix} E & F \\ F & G \end{pmatrix}$ and the parametrization matrix $\begin{pmatrix} \frac{\partial \varphi_i}{\partial \theta} & \frac{\partial \varphi_j}{\partial \theta} \\ \frac{\partial \varphi_i}{\partial \psi} & \frac{\partial \varphi_j}{\partial \psi} \end{pmatrix}$. Or we may directly calculate the energy, charge density and current especially at the boundary of the enclosed surface area curve c etc of a particular object such as the Taylor series like inverted cones approximation to this writer's present suggested Gabriel Trumpet Black Hole Toy Model [3] (We also have the modified new Kerr Toy model for the Black Hole but the present focus is my old toy model). In fact, the above description is only the matrix operation for the transformation between manifold and change of coordinates that may be referred to as the pullback of volume forms over the integration on the manifold or those of the related mathematics theories and computations etc [1].

Another interesting thing that this writer may want to emphasis is that as I have mentioned in my previous paper [3], there may be a magnetic monopole existing between the Riemann Zeta zeros, say ξ_i and ξ_j as the $\nabla \cdot B = 0$ or $\oint B \cdot dA = 0$ in an open (quantum) system. In fact, a magnetic monopole cannot exist itself, it should appear in pairs or the magnetic monopole must have its counter particle – magnetic anti-monopole. In reality, according to the dual super-conductor model, the existence of monopole & anti-monopole must lead to the appearance of dual quarks or an explanation to the confinement of quarks in terms of an electromagnetic dual theory of super-conductivity. In other words, my proposed (finite number of termed) Taylor series like inverted cone approximation to the Gabriel Trumpet Black Hole model implies furthermore the existence of the super-conducting phenomenon laying between the sandwiched (or the differenced) areas of those non-trivial Zeta zeros. In practice, the monopole (& anti-monopole) in the quantum perspective may be just known as the spin-ice while the real dual quarks is only the dual super-conductor models which posit the condensation of these magnetic monopoles in a superconductive state.

Therefore, this writer conclude that in theory, it is feasible we may finally have our superconducting electrical long distance wire which is free of resistance for our long distance electricity transmission etc. Actually, the production of such long distance superconducting electrical wires (with essential & necessary improvements) may be in the scope of engineering or even business and is out of the interested field with my present paper. In fact, the insulator may be needed to coat on the surface of such a superconducting electrical wire as a way to prevent human or animals from the high powered voltage electrical shocking.

In reality, this writer wants to note that in the case of gravitational waves (or perturbations) may affect the super-condensates and super-currents dynamics, i.e. superconductors and gravity or an anti-gravity devices. In its mirrored image inverse, under certain circumstances, a secondary gravitational field could be induced inside a super-conductor or a super-conducting quantum computer. These two facts may then constitute a kind of gravity-superconductor forward and backward philosophy. This writer will leave those detailed researches to the interested parties or readers as the main focus of the present paper is in the energy and battery of the proposed Trumpet Black Hole model such as the super-conducted long distance electrical wire but neither in the anti-gravity (or the gravitational-magnetism related nor the superconducted quantum computer etc.

On the other hand, if we assume a closed surface or the boundary of a surface without any edges, then for the same Maxwell equation $\nabla \cdot B = 0$ or $\oint B \cdot dA = 0$, this will imply the inward flux of the surface is just equals to the outward flux. To be precise, there have already been an equilibrium achieved between the virtual particle pairs popping in and out of the event horizon. That say, one of the quantum entangled pair particles will be held in the black hole while the others will fly out of it as the Hawking radiation.

To sum up from the above two feasible situations for a black hole, this writer may thus propose that the inside structure of a black hole (assume it is an open quantum system) may be far away more complexed than what we may think and observed. Actually, the black hole's event horizon may consist of a former habitable zone of an icy planet (with the existence of the spin-ice or the monopole and anti-monopole pairs inside a black-hole) that is responsible for her superconductivity. An in-sided black hole may transform gradually and finally end up with (while the other possibility is a singularity or) a high density of quarks or the so-called quark star as well as the up-raising properties of a superconductivity from the increasing in the dual paired of quarks and decreasing in the amount of the monopole & anti-monopole. At the same time, the outer black-hole; the outer boundary without edges or the event horizon is always in an equilibrium with the quantum entangled pairs of particles where one of the pairs will be caught and stayed inside the black hole and the other one of the pairs will be emitted from the black hole in the form of the Hawking radiation. The aforementioned proposed characteristics and phenomenon of a blackhole may thus be this writer's suggested way or the completed story of an evolution, conducting property and structure for my Trumpet toy modeled black-hole.

(N.B. (Source to Field): If one knows the total charge $Q_{enclosed}$ inside a volume, the outward flux of the Electric field ($\oint E.dA$) through the bounding surface must equal Q/ε_0

(Field to Source): If one measures the total flux passing through a closed bubble, one may mathematically determine exactly how much charge is trapped inside, regardless of how the charge's distributed.)

(N.B. This author has already simplified the surface area of the Trumpet black hole by the 3D cone. In reality, the true surface area of the Trumpet black hole is in fact:

$\int_a^b 2\pi (f(x)) \sqrt{1+[f'(x)]^2}\, dx$ or just as the case $\int_1^{11} 2\pi \left(\frac{1}{x}\right) \sqrt{1+\frac{1}{x^4}} dx$ for the first part of the Trumpet without the lateral part up to the high density singularity.)

(N.B. The Taylor Series of $\int_1^{11} 2\pi \left(\frac{1}{x}\right) \sqrt{1+\frac{1}{x^4}} dx$ is:

$$\int_1^{11} 2\pi \left(\frac{1}{x} + \frac{1}{2x^5} - \frac{1}{8x^9} + \frac{1}{16x^{13}} - \ldots \right) dx = \left[ln(x) + \frac{1}{2}\left(\frac{x^{-4}}{-4}\right) - \frac{1}{8}\left(\frac{x^{-8}}{-8}\right) + \frac{1}{16}\left(\frac{x^{-12}}{-12}\right) \right]_1^{11}$$

$= [2.3978867352 - (-0.114583333333)]* 2\pi$

$= 2.51247006853*2\pi = 15.7863150193$

which is integrable and can be evaluated & approximated once if we may truncated those tail terms as the aforementioned. Certainly, as this author has mentioned in his another paper, the volume of the Trumpet is finite and will not repeat its details. In fact, the object with infinite volume and finite surface area is impossible. The only interesting thing is when $f(x) = (-1/x(ln(x)^2)$ where the volume and the surface area will both be infinity if $x = 1$ and the integral of $f(x) = 1/ln(x)$.

(N.B. In fact, the equation for the Gabriel Trumpet may be considered as the acoustic impulse response that reacts to a sudden "impulse" of sound. One may even apply an impulse deformation model to find out how a complex surface may deform under force. By the way, with the simplified Trumpet black hole (or just the layered 3D cone) itself may be acted as a capacitor, filter (or actually the acoustic impulse response), together with the feedback (surrounded by a reflective surface like the spherical mirror or a massive field) and the amplifier (or a rotating Kerr Black hole) building blocks, one may then establish the corresponding signal control system for the respective frequencies like removing some of the engineering noise or damping some unwanted oscillations & the highly magnetized, rotating neutron stars that emits beams of electromagnetic radiation from their poles will be considered as the Pulsars for the signal coordinator or the Green waves as a method of optimization for my presently proposed universe-spaced signal control system etc.)

(N.B. The inverse of the above universe-space signal control system may be referred to how we may guess back what the original space object(s) may be from those distorted signal we have received from the appliance(s) or methods just like the Earth-based telescope(s), radar imaging, satellite, computer software algorithm like CLEAN, Inverse Compton Emission, trajectory Optimization, Attitude Control & Precision Docking etc)

Actually, for the famous Schwarzschild Equation as shown below:

$$\left(1-\frac{r_s}{r}\right)c^2dt^2-\frac{1}{\left(1-\frac{r_s}{r}\right)}dr^2-r^2d\Omega^2$$

If we let $\left(1-\frac{r_s}{r}\right)$ as a complex variable, z, then the above Schwarzschild Equation will be turned into the following:

$$(z)c^2dt^2-\frac{1}{(z)}dr^2-r^2d\Omega^2$$

which will then eliminate the singularity at the end of the black hole as if we employ the complex cylindrical coordinates or complex spherical coordinates from a complex Schwarzschild metric for the contour integrals, then by the Cauchy-Goursat Theorem:

$$\oint_{|z|=r} z^{-1}dz = I\int_0^{2\pi} e^0 d\theta = 2\pi I$$

and

$$\oint_{|z|=r} zdz = 0$$

The Schwarzschild Equation will be reduced into:

$$\frac{1}{(z)}dr^2-r^2d\Omega^2$$

or

$$2\pi Idr^2-r^2d\Omega^2$$

where the singularity at the centre of the black hole will thus be disappeared. All we need is just to solve the above reduced Schwarzschild equation.

If we go ahead for a further step and replace the variable z by the zeta function "zeta z" like my proposed Trumpet Black Hole Toy model, we may get the following equation:

$$2\pi Idr^2-r^2d\Omega^2$$

where n = 1,2,3…. which is just the number of the zeta function's roots. These singularities may be just the entries or exits for another universes. Therefore, the theoretical multiverse may really exist.

In much of the same way, we may have

$$2\pi Idr^2-r^2d\Omega^2$$

If we subtract the above two equations with each other and equate it to zero, we may get:

$$2n\pi I\left(c^2dt^2-dr^2\right)=0$$

$$c^2dt^2=dr^2$$

Integrate both side, we may then have:

$$c^2\,t^2 = r^2 + K$$

When r = 0, t = 0, hence K = 0,
thus $c^2\,t^2 = r^2$ or r = ±ct
or time $t = \pm r/c$.

In other words, we may have actually solved the Schwarzschild equation for myself proposed Trumpet Blackhole Model. It is true that we may continue the computational procedure by some mathematical software like the Maple Soft to further solve the general solution for the Einstein's special relativity. By employing the time dilation equation and the length contraction equation, we may finally get the following equation:

$$\Delta t_0=\frac{\gamma r_0}{c}$$

where r_0 is the original length of the Black Hole radius before length contraction or time dilation, $\gamma=\sqrt{1-\frac{v^2}{c^2}}$ and c is the speed of the light.

In practice, there is a simple way to find out the integral of the Schwarzschild geodesics equations like the following:

$$\varphi(r_s,b,R_{max})=\int^{R_{max}}\frac{dr}{r\sqrt{\frac{1}{b^2}-\left(1-\frac{r_s}{r}\right)\frac{1}{r^2}}}$$

Suppose $Z=\left(1-\frac{r_s}{r}\right)$ and let $\frac{1}{b^2}-\left(\frac{\sqrt{Z}}{r}\right)^2=K^2$ (by the Pythagoras Theorem), then we may have:

$$
\begin{aligned}
\varphi(r_s, b, R_{max}) &= \int_0^{R_{max}} \frac{dr}{r^2\sqrt{k^2}} \\
&= \left(\frac{1}{k}\right)\left(-\frac{1}{R_{max}} + \frac{1}{ln1}\right) \\
&= \frac{1}{k}\left[\frac{1}{z'}\right]_0^{Rmax} \\
&= \frac{1}{k}\int_0^{R_{max}} \frac{1}{z'} dz'
\end{aligned}
$$

But the line integral of $\frac{1}{k} \to \oint_{z'=|R_{max}|} \frac{1}{z'} dz' = \frac{1}{k}(2n\pi I)$ for n = 0, 1, 2...

which is just the equation for the bending light paths or the rings of beams by the strong gravity of my proposed Trumpet Black hole toy model [3] (that includes details about the elementary ratio of the zeta zeros' layered electrical potential) with reference to the integral of the Schwarz-child geodesics in the Theory of General Relativity and these rings are also in practice a path of the (or some) concentric circle(s).

(N.B. In the sense of signal processing engineering, we may in fact from the above line contour integral to further define a complex Fourier series by

$$\lim_{R_{\max}\to\infty} \sum_{N=0}^{R_{max}} \frac{e^{In\theta}}{z^N} = \lim_{R_{\max}\to\infty} \oint_{z'=|R_{max}|} \frac{e^{In\theta}}{(z')^N} dz'$$

which will give either a zero or

(2πI)*(Residue at the pole) according to the N = 1 or otherwise but this is out of the scope of the present study in my proposed Trumpet Black Hole Toy Model. I will leave such related topic in the complex Fourier series and the associated signal filter together with the digital signal control system for those interested parties to have a deep and wide research.)

We may need to redefine the generalized metric (or actually the complex Schwarzschild metric); metric tensor or the matrix field G = g = (g_{ij}) satisfying the complex numbered cylindrical (or spherical) coordinate system (for the complex variable $z = x + yI = re^{I\theta}$) or a semi-Cartesian 3D with semi-complex 3D polar coordinate, we may have the over-ride the Laplace operator

$[\nabla](V_{ij}^{\alpha})$ w.r.t. the partial differentiation of time t and x^1, x^2, x^3 axis for the 4-dimensional positional vector with the upper index α plus the matrix field entries $[\nabla](V_{ij}^{\alpha}) = [J]_{ij}$ or the matrix field G = g = J^xJ = $[\nabla](V_{ij}^{\alpha})x[\nabla](V_{ij}^{\alpha}) = [[\nabla](V_{ij}^{\alpha})x[\nabla](V_{ij})$ where x = -1 (the inverse for the Affine Tensor, x = T for the transformation to be orthogonal) for the complex variable z where J is the corresponding Jacobian matrix for the transformation [3] between the 3D Cartesian-complex polar coordinate. Moreover, we should assume the operation $[\nabla](V_{ij}^{\alpha})x[\nabla](V_{ij}^{\alpha})] = [\nabla](V_{ij}^{\alpha})$ $x[\nabla](V_{ij}^{\alpha})$ is practically equivalent to each other. In addition, this writer likes to use J = $[\nabla](V_{ij}^{\alpha})$ as it seems to be more accurately and specially in depicting the actually partial differentiation context or component behind $[\nabla](V_{ij}^{\alpha})$ which is significant in the Einstein physics or index notation. In practice, $[\nabla](V_{ij}^{\alpha})$ has more symbolic implication behind than the symbol J. On the contrary, the symbol Jacobian J which may show no hints about the partial differentiation that actually has been applied. By the way, the two symbols J and $[\nabla](V_{ij}^{\alpha})$ are in fact equivalent in this writer's sense.) Hence, this writer may NOT have enough resources for the such large project as this paper is just an empirical research and I will leave this redefinition and the rest related computation to those interested Mathematical (or Physics) research institute(s) or universities etc.)

By the way, we may have:

$2(\gamma r_0)^2 = d r^2$

i.e. $r^2 = 2\gamma^2 r_0^{\ 2} + k$

Or we may have the proposed solution to the Einstein Field Equation for the Trumpet Black Hole Toy Model as below:

$$
\begin{pmatrix}
1 - \frac{r_s}{\sqrt{2\gamma^2 r_0^{\ 2} \ \ k}} & 0 & 0 & 0 \\
0 & \frac{1}{1 - \frac{r_s}{\sqrt{2\gamma^2 r_0^{\ 2} \ \ k}}} & 0 & 0 \\
0 & 0 & -2\gamma^2 r_0^{\ 2} \ \ k & 0 \\
0 & 0 & 0 & \left(-2\gamma^2 r_0^{\ 2} \ \ k\right) sin^2\theta
\end{pmatrix}
$$

In brief, we may solve the aforementioned reduced Schwarzschild equation in an easier way. Therefore it is an interesting topic in the black-hole astro-physics for a further wider and an in-depth study or research such as

$$\mathbf{R_{\mu\nu} = 0.}$$

The Philosophy Behind the Stokes' Theorem & Conclusions

In practice, there may be an analogy between the Stoke's Theorem and the human intelligence. As we may know intuitively that:

Reasoning + Perception (Cognitive Feelings) = Action.

This is because we have both "For" reasons and "Against" reasons about an issue. Then we may deduce our position according to the "logical deduction" behind those "For" and "Against" reasons. At the same time, when one is handling a public issue or affairs, we may still have the associated "Positive" and "Negative" cognitive feelings or perception with reference to the public propagation, previous education, known moral rules as well as the surrounding environmental people's comments etc. Then all of these two factor (reasoning and perception) s' rationales will help a person to determine what may be his/her next step or action. Thus, with the aforementioned justifications, this writer proposes that both the reasoning together with the perception (cognitive feeling) will induce the action just like the Fleming 's right hand rule in the electro-magnetism of Physics that: A moving charge under the electric flux line will induce current. In fact, the Stoke's theorem is just used to reduce high dimensional integral into those lower one. Green's Theorem and the Divergence Theorem [1] are actually the low dimensional Stoke's theorem which may be applied directly in the Maxwell's Electricity and Magnetism Equations. Therefore, this writer may conclude that there is actually an analogical relationship staying between the Stoke's Theorem and the human intelligence or this is just the philosophy behind the Stoke's Theorem.

In reality, when we are talking more about those applications, this author suggests one may have the Trumpet black hole modeled focal point electronic gun or even use the forward and vice versa part of the Trumpet black hole model acting as the deep universe (at least in the Solar system) antenna for the transmission of the communication data or signal(s), or the so-called the "Deep Space Universe Internet" etc.

To conclude, we may treat the gap between the two layered zeta zeros as a capacitor or one may compute the energy stored by the Stoke's Theorem. Actually, one may do so under the help of the mathematical software like the Maple Soft. In reality, there are lots of applications with the force induced by the electric field as well as the superconducting properties being investigated empirically. In addition, we have NOT only the physical achievements among the Black Hole Toy Model layers, but there is also a philosophical connection between the Stoke's Theorem and the human intelligence as we may have well mentioned already before. Thus, this writer proposes that the present research in the Stoke's Theorem and battery energy is in fact an interesting one and NOT so much people may have done that.

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