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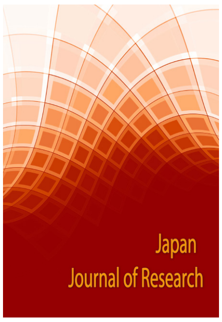


# From Newton's First Law to the Motion of Celestial Bodies

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**Abstract**

*Newton's original work "Principles" was written in Latin. An English version translated two years after his death changed the first law from "in directum" to "in a right line." Later generations think that only static or uniform straight line motion is inertia, while excluding rotational motion. This article proposes that the gravitational potential energy of a planet's revolution is equal to its centrifugal kinetic energy (mυ2). The work done by an object is a process quantity, but kinetic energy is a state variable that can be divided into instantaneous kinetic energy and average kinetic energy. Uniform motion does not do work, and instantaneous kinetic energy in variable speed motion does not also do work, only the average kinetic energy of variable speed motion does work. Only the energy between work and the average kinetic energy of variable speed motion can be converted into each other. We have clarified the relationship between work and kinetic energy, completed the transition of the kinetic energy formula at low and high speeds, achieved the unification of the kinetic energy formula and the mass energy relationship formula, and explained the √2 factor in the escape velocity..*

## Reexamine Newton's First Law

Newton's First Law (NFL) is a very rare but striking physical law that is only presented in textual form. NFL first appeared in the "Mathematical Principles of Natural Philosophy" (Principia). Although "Principles" is a treasure of British culture, it is written in Latin. The first two Latin versions appeared in 1687 and 1713, and their descriptions of NFL were roughly the same. They read

*"Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare. (emphasis added)"*. [1]

Due to the pure textual form of Latin, it has affected people's understanding of NFL. This situation makes it difficult for future generations to reveal the true meaning of NFL. The Latin literal meaning of the word " movendi uniformiter in directum " that describe movement is " move uniformly in the direction ". We analyze that its significance should not exclude "straight line direction", but it does not mean it is limited to "straight line direction". That is to say, in the earliest two Latin versions, regarding NFL, Newton only proposed in stationary or move uniformly in the direction.

Two years after Newton's death, mathematician Motte published the first English translation of the "Principles". Among them, NFL reads

*"Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed there on"*.[1]

Obviously, regarding the description of motion, Mott translated the original " in directum " as "in a right line", strictly limiting the motion state to a motion in right line. Although the title of Newton's "Principles" contains the words "Mathematical Principles," Newton mainly discussed physical problems. Mott is a mathematician, not a physicist. Has his translation bound or altered Newton's original meaning? The translation was published two years after Newton's death, but, of course, Newton did not see it.

Let's analyze this issue from another perspective. As is well known, a problem that Newton never solved throughout his life was the initial driving force of celestial body motion rather than the current motion force when studying the revolution and rotation of planets. Obviously, the current motion of celestial bodies is regarded by Newton as inertia and does not require external forces. Perhaps in Newton's thinking, the revolution and the rotation of celestial bodies were already

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included in his first law. By this, we speculate that the original meaning of NFL should be that all objects should remain stationary or maintain their original uniform motion state, unless it is compelled to change that state by forces impressed thereon.

The original motion state can be uniform straight-line motion or uniform circular motion. By this way, the property of objects maintaining uniform circular motion can be incorporated into inertia. In today's rapidly developing society, the phenomenon of rotational inertia is ubiquitous. Even if the power is turned off, a fast-rotating grinder can still rotate for a long time. It is difficult for high-speed vehicles to immediately stop the rotation of their wheels when suddenly braking. Any object with mass has moment of inertia when rotating, and it is difficult to stop immediately when rotating rapidly. Who can say that these rotations that cannot be stopped immediately do not belong to inertia? If uniform rotation is included in the category of inertia, whether considering the Earth's revolution or rotation, we will be in an inertial frame.

Influenced by Mott's translation, people's understanding of NFL is still limited to stationary and uniform straight-line motion. Compared to Newton's second and third laws, it is obvious that the pattern of the translated NFL is too small. In fact, stationary and uniform straight-line motion are not absolute realities in the entire universe. If NFL is understood only in terms of stationary and uniform straight-line motion, then it is only approximately applicable in the small living areas near us. The motion of celestial bodies is an important aspect of Newton's kinematic thinking, and such a narrow understanding (uniform straight-line motion) clearly does not conform to Newton's ideas.

## Centrifugal force in rotational motion

A typical example of Newton's study of rotational motion is his bucket experiment. He proposed this idea experiment to prove that absolute rotation can be distinguished from relative motion. The experiment is simple, it includes a bucket, some water, and a long rope, but Newton spent two-thirds of the space describing it and its results. [2] Newton realized that any liquid rotating in a container would have a parabolic shape and described this experiment in 1689. The higher the rotational speed, the deeper the parabolic meniscus. A same phenomenon is observed when the liquid level is bent while stirring coffee in a cup. [3]. As shown in Figure 1, the curvature of the water surface is independent of the motion of the water relative to the bucket wall. That is to say, regardless of whether the bucket rotates or not, as long as the water in the bucket rotates rapidly, the water surface will produce a parabolic meniscus. We are quoting this classic example here to remind everyone that centrifugal force is generated when the water in the bucket rotates around the center, and centrifugal force is the essence of this experimental phenomenon. Any object with a mass of m, at a linear velocity $\upsilon$, when performing circular motion, a centrifugal force $F$ is generated. If an object moves in a uniform circular motion with a radius $R$, the centrifugal force $F$ has the following accurate mathematical relationship

$$F = \frac{m\upsilon^2}{R} \qquad (1)$$

In the bucket experiment, no centripetal force was applied to the water at the center of the bucket. Therefore, it can be proven that centrifugal force is a special independent force, not a reaction force of centripetal force. The centrifugal force generated by the rotation of water is counteracted by the obstruction of the barrel wall. When there is no barrel wall, the rotation of water still generates centrifugal force, such as eddies in rivers, lakes, and seas. The direction of centrifugal force is the direction of curvature radius, which is always perpendicular to the direction of object motion. Centrifugal force does not change the magnitude of velocity, only the direction of velocity. When the curvature radius $R \to \infty$ and $F \to 0$, it can be regarded as a uniform straight-line motion. Therefore, uniform straight-line motion is a special case of uniform circular motion at $R \to \infty$. Figure 2 shows a hammer throw player forcefully pulling a hammer and rotating it around the common center of mass of the person and the ball. Both person and hammer generate an outward centrifugal force. When the person releases his hands, the hammer flies out along the tangent direction.

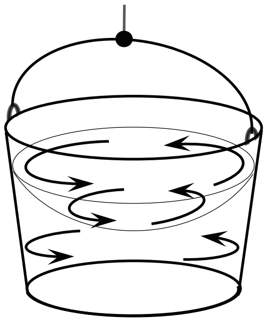

***Figure 1.*** *Newton's bucket experiment showed that rotating water generates centrifugal force.*

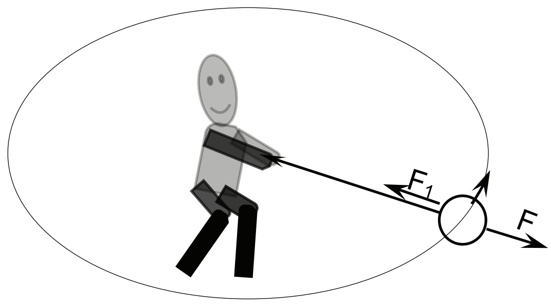


***Figure 2.*** *The centrifugal force generated by the hammer in circular motion is F, and the centripetal force F1 is the reaction force of F.*

## General Newton's first law and the centrifugal kinetic energy of celestial motion

If the expansion effect of cosmic evolution is not considered, then all celestial bodies can be approximated as moving in uniform circular or elliptical orbits at different levels. Uniform straight-line motion can be regarded as a uniform circular motion when the curvature radius tends towards infinity. The stationary state of an object is simply a relative rest state when it moves uniformly in a circular motion with its surrounding environment. For example, as humans and our surrounding environment, including mountains, lakes, trees, houses, etc., move in a uniform circular motion with the earth, we feel like we are stationary. Therefore, stationary and uniform straight-line motion are only special cases to uniform circular motion. The space station orbits around the Earth, and the gravitational force generated by the Earth on the astronauts inside is equal to their centrifugal force. Their bodies are floating and do not feel any directional force. The examples mentioned above for uniform circular motion are all inertial motion, which may be Newton's original intention about the first law, or an extension or modification of NFL by this article. It doesn't matter which situation it belongs to. Importantly, we can no longer be bound by the current interpretation of the translated NFL, and based on that the uniform circular motion is inertia, we can study the motion of all celestial bodies even entire universe. To distinguish, the NFL containing rotation motion is referred to as generalized NFL, abbreviated as GNFL.

Based on GNFL and ignoring the interference of other planets, we can analyze the Earth's orbit around the sun using a simple two body problem. Using $M$ and $m$ represent the masses of the Sun and Earth, respectively, and $M \gg m$. Let's first approximate the Earth's orbit around the sun as a uniform circular motion. The Earth is subjected to the gravitational force of the Sun as $F_1$, with $F_1$ pointing towards the center of the circle orbit.

$$F_1 = \frac{GMm}{R^2} \qquad (2)$$

where $G$ is the constant of universal gravitation, and $R$ is the radius of the Earth's revolution. The centrifugal force $F$ represented by equation (1) of the Earth's revolution is in the radial direction. $F_1$ and $F$ are opposite in direction, equal in size, and mutually balanced. Their directions are all perpendicular to the direction of motion, so the magnitude of their motion speed remains unchanged. That is, the acceleration in the direction of motion is zero. Now we can say that the Earth's revolution is a never-ending inertial motion. By $F_1 = F$, we have the following equation

$$\frac{GMm}{R^2} = \frac{m\upsilon^2}{R} \qquad (3)$$

or

$$\frac{GMm}{R} = m\upsilon^2 \qquad (4)$$

On the left side of equation (4) is the gravitational potential energy $V$ generated by the sun on the Earth. The potential energy usually takes a negative value, i.e

$$V = -\frac{GMm}{R} \qquad (5)$$

The $m\upsilon^2$ on the right side of equation (4) has a dimension of energy. By the Encyclopædia Britannica, “kinetic energy is a form of energy that an object or a particle has by reason of its motion”. According to dimensions and definitions, the $m\upsilon^2$ can be used as a candidate calculation formula for kinetic energy. Namely

$$E = m\upsilon^2 \qquad (6)$$

To distinguish it from the current kinetic energy calculation formula $\frac{1}{2}mv^2$, we temporarily refer to equation (6) here as the centrifugal kinetic energy generated by the Earth's orbital motion. In this article, $v$ is used to represent variable speed, and $\upsilon$ represents a constant speed for maintaining inertial motion.

The Earth's orbit is approximated as a circular orbit, and the correctness of equation (4) is verified using well-known data. The mass of the Earth is $m$ =5.965 x $10^{24}$ kg, the average distance between the Sun and Earth is $R$ =1.496 x $10^{11}$ m, the average revolution speed is 2.978 x $10^4$ m/s, the mass of the Sun is $M$ =1.989 x $10^{30}$ kg, and the gravitational constant G = 6.67 x $10^{-11}$ N · m²/kg².

Calculating the potential energy $V$ of the Earth in the solar gravitational field using equation (5) is

$$V = -\frac{GMm}{R} = -\frac{6.67\times10^{-11}\times1.989\times10^{30}\times5.965\times10^{24}}{1.496\times10^{11}} = -5.29\times10^{33}\,(J) \qquad (7)$$

Calculating the centrifugal kinetic energy of the Earth's orbital motion using equation (6) is

$$E = m\upsilon^2 = 5.965\times10^{24}\times(2.978\times10^4)^2 = 5.29\times10^{33}\,(J) \qquad (8)$$

The calculation results by equation (7) and equation (8) are exactly the same numerically, and only one negative sign is missing, indicating that the equation (4) we obtained is completely correct. The centrifugal kinetic energy and potential energy of the Earth's revolution are numerically equal. The Earth approximately moves in a uniform circular motion around the Sun, which belongs to inertial motion and not doing external work or being affected by other external forces. If the potential energy is negative and the influence of other planets is ignored, the total energy (potential energy + centrifugal kinetic energy) of the Earth's orbital motion is 0. Therefore, it corresponds to an energy conservation system. If the calculation accords to the current kinetic energy formula, the kinetic energy of the Earth is only half of the potential energy or centrifugal kinetic energy, and is not an energy conservation system.

Equations (4), (5), and (6), although derived from the Earth's revolution, can apply to all planets in the solar system. Table 1 lists the ratio of centrifugal kinetic energy and mass for the eight major planets in the solar system during orbital motion calculated by equation (10), and average orbital velocity. The semi major axis a of the orbit in Table 1 replaces the radius $R$.

By equation (4), the following equation can also be obtained:

$$GM = \upsilon^2 \mathrm{R} \qquad (9)$$

Equation (9) indicates that the product of the orbital radius and the square of the orbital velocity for all planets in the solar system is a constant, which is the product of the mass of the sun and the gravitational constant. Equation (4) can also be written as

$$\frac{GM}{R} = \upsilon^2 \qquad (10)$$

Comparing equations (4), (9), and (10), it can be seen that, $\upsilon$ is the only factor that balances the gravitational pull of central celestial bodies. According to equation (10)

$$\upsilon = \sqrt{\frac{GM}{R}} \qquad (11)$$

Table 1 lists the orbital velocities ($\upsilon$) of the eight major planets in the solar system calculated by equation (11), and $\upsilon$’ is the literature value. [4] The $\upsilon$ calculated to $\upsilon$’ completely match within the range of significant digits.

***Table 1.*** *The ratio of centrifugal kinetic energy ($m\upsilon^2$) with mass of the Eight Major Planets in the Solar System*

| Planet | | Mercury | Venus | Earth | Mars | Jupiter | Saturn | Uranus | Neptune |
|---|---|---|---|---|---|---|---|---|---|
| $a$ | $\times10^{11}$ m | 0.579092 | 1.082095 | 1.495983 | 2.279438 | 7.783408 | 14.26666 | 28.70658 | 44.98396 |
| $E/m$ | km²/s² | 2291.732 | 1226.440 | 887.1255 | 582.2156 | 170.5068 | 93.02275 | 46.23067 | 29.50217 |
| $\upsilon$ | km/s | 47.872 | 35.021 | 29.785 | 24.129 | 13.058 | 9.645 | 6.799 | 5.432 |
| $\upsilon'$ | km/s | 47.360 | 35.020 | 29.783 | 24.077 | 13.050 | 9.639 | 6.795 | 5.432 |

If the period of orbital motion is represented by *T*, then $v = 2\pi R/T$. Substituting $2\pi R/T$ into equation (9), yields

$$\frac{GM}{4\pi^2} = \frac{R^3}{T^2} \qquad (12)$$

Equation (12) is the famous Kepler's third law. A circular orbit can be seen as a special case of an elliptical orbit when the major and minor axes are equal.

## Work-kinetic energy theorem and centrifugal kinetic energy of uniformly moving objects

The definitions of work and kinetic energy were first proposed by French physicist Coriolis (Gustave Gaspard de, 1792-1843) in 1829. [5] By the Encyclopædia Britannica,*"If work, which transfers energy, is done on an object by applying a net force, the object speeds up and thereby gains kinetic energy. Translational kinetic energy of a body, (EK) is equal to one-half the product of its mass m and the square of its velocity (v)."*

$$E_K = \frac{1}{2}mv^2 \qquad (13)$$

"*The formula (13) is valid only for low to relatively high speeds; for extremely high-speed particles it yields values that are too small.*"

Equation (13) is obtained from the work-kinetic energy theorem.

In 2006, American physics teacher Kamela, M. pointed out:

"The work-kinetic energy theorem is one of the fundamental and more subtle concepts students see in their freshman physics course. …… It is challenging for students to appreciate the work-kinetic energy theorem in the case of a variable force acting over some displacement." [6]

Although the proof of the work-kinetic energy theorem is based on the conservation of energy, the physical meanings of work and kinetic energy are clearly different. Kinetic energy is a state variable that describes a moving object or particle, while work is a process variable. Thermodynamically, work is not a state function. During the process of doing work, the kinetic energy state of an object changes ceaseless with velocity. Doing work is completed throughout the entire process of state change. According to GNFL, objects in uniform or inertial motion do not do work, therefore the work-kinetic energy theorem cannot be used to calculate and prove the kinetic energy of objects in uniform motion. Kinetic energy is a function of velocity, and in variable speed motion, there are instantaneous velocity and average velocity. Therefore, in variable speed motion, we should also distinguish between instantaneous kinetic energy and average kinetic energy.

According to the definition of work, an object must move a certain distance along the direction of the force in order to do work, and magnitude of the work is determined by the product of the two. In uniform motion, there is distance but no force, so no work is done. In variable speed motion, when instantaneous kinetic energy is used, there is no duration, that is, there is force but no distance, so instantaneous kinetic energy cannot be converted into work; When average kinetic energy is used, there is duration, that is, there are both force and distance, so it can be converted into work. Therefore, in variable speed motion, only the average kinetic energy of an object can be converted into work, following the law of energy conservation.

Next, we will analyze the proof process of the work-kinetic energy theorem and find a possible expression for the kinetic energy of a uniformly moving object. The work done on an object can be calculated by multiplying the distance of displacement and the applied force, i.e

$$W = F{\cdot}l \qquad (14)$$

When a moving object does work, there will definitely be a change in velocity. l is the displacement distance under variable speed, and *F* can be a variable force or a constant force. We choose a most typical and simplest motion with uniformly varying velocity for discussion. In a process of uniformly varying velocity, l is equal to the product of average speed and time, and *F* is a constant force.

Let's first discuss the case where the velocity varies uniformly between *0* and *v*. Figure 3A) shows the process of an object accelerating uniformly from rest ($v_0 = 0$) to a velocity *v*; Figure 3B) shows the process of an object decelerating uniformly from velocity *v* to rest. In both processes, the displacement distance l is equal to the product of time *t* and average velocity ½*v*, that is, it is equal to the area of the triangle in the graphs. Namely

$$l = \tfrac{1}{2}vt \qquad (15)$$

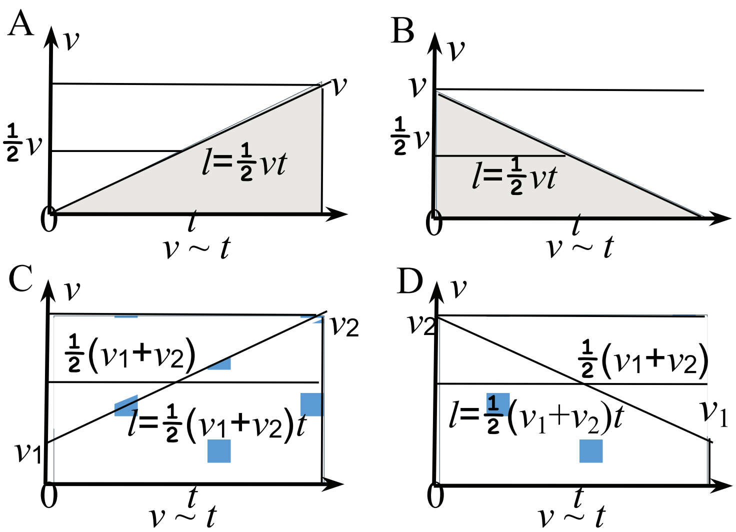


***Figure 3.*** *The displacement distance of a uniform speed change process depends on the average speed; A) A uniform acceleration process starting from rest; B) The process of uniform deceleration until stationary; C) The uniform acceleration process from v1 to v2; D) The uniform deceleration process from v2 to v1.*

If an object is moving at a constant speed $\upsilon$, the displacement distance is

$$l = \upsilon t \qquad (16)$$

The distance of uniform motion represented by equation (16) is equivalent to the area of a rectangle in Figure 3, which is exactly twice the distance of motion with uniformly varying velocity.

The process of uniform acceleration involves constant force doing work. According to Newton's second law, the force is

$$F = ma = m\frac{v}{t}. \qquad (17)$$

Work done within time t is

$$W = Fl = m½vt = ½mv^2 \qquad (18)$$

Equation (18) is also the work-kinetic energy theorem represented by equation (13). We notice by the above proof that W is the energy conversion of the entire process of an object from rest to velocity $v$. The instantaneous kinetic energy state at velocity $v$ is only the starting or ending point of the process of doing work. Obviously, $W$ is not equal to the instantaneous kinetic energy of the object at rest, nor equal to the instantaneous kinetic energy of the object at its maximum velocity $v$. The displacement distance of an object during the process of doing work is calculated by the average velocity $½v$, while constant force is calculated based on the maximum velocity $v$. Therefore, the work $½mv^2$ is the kinetic energy at the root mean square velocity, i.e. the square root of the product of the average velocity and the maximum velocity, $(1/\sqrt{2}v)$. Note that root mean square velocity is not the average velocity.

By the law of conservation of energy, if a work $½mv^2$ is done on an object, the work $½mv^2$ can accelerate the object and converts it all into its average kinetic energy. The velocity of the object can only reach the root mean square velocity, $1/\sqrt{2}v$. The speed v in current formula of kinetic energy was actually an overestimation due to a misunderstanding for the work-kinetic energy theorem. That is to say, if we do work of $½mv^2$ but cannot reach the velocity $v$ of the object at all.

For the relationship between displacement distance and velocity during the uniform variation of velocity between $v_1$ and $v_2$, the process of uniform acceleration of velocity from $v_1$ to $v_2$ is shown in Figure 3C), and the process of uniform deceleration of velocity from $v_2$ to $v_1$ is shown in Figure 3D). In both cases, the displacement distance l is equal to the product of average velocity $½(v_1+v_2)$ and time t, and in the graph, it is equal to the shaded area from the diagonal to the bottom (triangle + rectangle). Namely

$$l = ½(v_1 + v_2)t \qquad (19)$$

The process of uniform acceleration involves constant force doing work. According to Newton's second law, the force

$$F = ma = m\frac{v_2 - v_1}{t}. \qquad (20)$$

Work done within time t is

$$W = Fl = m\frac{v_2 - v_1}{t}\cdot ½(v_1 + v_2)t = ½m(v_2^2 - v_1^2) \qquad (21)$$

The work done during the uniform acceleration process from $v_1$ to $v_2$ is neither equal to the instantaneous kinetic energy of the object at the lowest speed $v_1$ nor at the highest speed $v_2$, but represents the kinetic of the object at the root mean square velocity $\sqrt{\frac{v_2^2 - v_1^2}{2}}$

The $v_1$ and $v_2$ can also be regarded as two velocity states that accelerate from rest to $v_1$ and $v_2$, respectively.

$$W_1 = ½mv_1^2; \quad W_2 = ½mv_2^2; \quad (22)$$

where $W_1$ represents the work done on object $m$ during the process of uniform acceleration from rest to velocity $v_1$, which is also equal to the average kinetic energy of the object during the process, or is equivalent to the instantaneous kinetic energy at root mean square velocity of $1/\sqrt{2}\ v_1$; $W_2$ represents the work done on object $m$ during the process of uniform acceleration from rest to velocity $v_2$, which is also equal to the average kinetic energy of the object during the process, or is equivalent to the instantaneous kinetic energy at a root mean square velocity of $1/\sqrt{2}\ v_2$. Because work and kinetic energy are both scalars, the work done on an object during the uniform acceleration process from $v_1$ to $v_2$ is

$$W = W_2 - W_1 = ½mv_2^2 - ½mv_1^2 = ½m\left(v_2^2 - v_1^2\right) \qquad (23)$$

where work $W$ is equal to the average kinetic energy of the object during the process from $v_1$ to $v_2$, or is equal to the instantaneous kinetic energy at the root mean square velocity, $\sqrt{\frac{v_2^2 - v_1^2}{2}}$

The proof process of the relationship between all work and the kinetic energy of an object mentioned above indicates that the work done during a uniform acceleration process is equal to the average kinetic energy of the object during the process, or is equal to the instantaneous kinetic energy at a root mean square velocity of $1/\sqrt{2}\ v$ or $\frac{1}{\sqrt{2}}\sqrt{v_2^2 - v_1^2}$. That is to say, the work done during the variable speed process $W = ½mv^2$ is not equal to the instantaneous kinetic energy of the object at the instantaneous velocity $v$, let alone the kinetic energy at a constant velocity $\upsilon$. It seems that the instantaneous kinetic energy of an object at an instantaneous velocity ($v$) is $mv^2$. we previously proposed that the centrifugal kinetic energy of an object is $mv^2$ when it is in uniform circular motion. Is the kinetic energy $mv^2$ the same as the centrifugal kinetic energy $m\upsilon^2$? Next, we will discuss deceleration and acceleration separately.

## Deceleration

During a process of variable speed, work only transforms with the average kinetic energy and follows the conservation of energy. The starting speed during a deceleration process is the highest speed, can also be the original constant speed, that is, it is the speed that the object has already reached at the starting point, $v = \upsilon$. The work done by the object in the process is equal to the average kinetic energy of the object, i.e

$$W = ½mv^2 = ½m\upsilon^2 \qquad (24)$$

Note that equation (24) represents that the average kinetic energy of an object is equal to the work converted during the deceleration process, not its instantaneous kinetic energy at the constant speed $\upsilon$ or instantaneous velocity v.

## Acceleration

For an acceleration process, the work done on an object can only be converted into its average kinetic energy $½mv^2$. When the instantaneous kinetic energy of the object reaches its average kinetic energy, its velocity only reaches its root mean square velocity $1/\sqrt{2}\ v$. The v is a maximum speed that the object has not yet reached when done work $½mv^2$ on the object.

By the conservation of energy, doing work $½mv^2$ on an object, the objects can only reach a uniform speed ($\upsilon$) or its root mean square velocity ($1/\sqrt{2}\ v$). Namely

$$\frac{1}{\sqrt{2}}v = \upsilon \ or \ v = \sqrt{2}\,\upsilon \qquad (25)$$

.

Substituting equation (25) into the current kinetic energy formula (13), yields

$$W = ½mv^2 = m\upsilon^2 \qquad (26)$$

Equation (26) indicates that if work is done on an object to increase it to a uniform velocity $\upsilon$, when understanding and calculating the work-kinetic energy theorem based on current theories, it requires $v > \upsilon$. In fact, during the acceleration process, the instantaneous kinetic energy of the object at v did not reach it, but only reached its average kinetic energy, and the instantaneous velocity only reached the root mean square velocity ($1/\sqrt{2}\ v$) at the average kinetic energy ($\frac{1}{2}mv^2$). The difference in velocity between the two kinetic energy formulas is a factor of √2.

## A √2 factor encountered in calculating escape velocity

There is a difference of √2 factor between the two velocities in the above two kinetic energy formulas, which has been discovered during the launch of artificial satellites or spacecraft. The flight speed around the Earth is calculated based on equation (11), which does not involve the kinetic energy formula. The speed of flying around the Earth is the true speed, which is the inertial speed

$$\upsilon_I = \sqrt{\frac{GM}{R}} = \sqrt{\frac{5.965\times 6.6726\times 10^{13}}{6.371\times 10^6}} = \sqrt{62.4738\times 10^6} = 7.904\times 10^3 \text{ m/s} \quad (27)$$

where $\upsilon_I$ also known as the first cosmic velocity or circumferential velocity, it is the speed at which an object on the Earth undergoes uniform circular motion and is the minimum speed required for weightlessness on the ground. So how fast do objects on the ground need to accelerate in order to break free from Earth's gravity and fly out of the Earth?

By equation (9), the product of the square of the revolution speed and the radius of the orbit is a constant. The larger the radius, the lower the speed. That is to say, if only the launch radius (altitude) is increased, the speed remains unchanged or slightly exceeds 7.9 km/s, then the object can fly away from the Earth. But tests and experience tell us that when launching flying objects from Earth, in order to fly away from Earth, it is necessary to reach the second cosmic velocity $v_{II}$

$$v_{II} = \sqrt{2}\upsilon_I = \sqrt{2}\times 7.904\times 10^3 m/s = 1.118\times 10^4 m/s \doteq 11.2 \text{ km/s} \quad (28)$$

Because the speed of launching a flying object is based on the kinetic energy formula $\frac{1}{2}mv^2$, the work done on the flying object only make it reach the average kinetic energy, not the instantaneous kinetic energy at the highest speed $v$. That is to say, the instantaneous velocity of the flying object only reaches its root mean square velocity, $v/\sqrt{2}$, rather than the maximum velocity $v$. If a flying object with a launch mass of $m$ moves uniformly around the Earth in a circular motion or flies out of the Earth, it must meet

$$\frac{v_{II}}{\sqrt{2}} \geq \upsilon_I . \quad (29)$$

Equation (29) indicates that if we want to truly reach the first cosmic velocity ($\upsilon_I$), we must multiply it by a factor (√2) when calculating it using the work-kinetic energy formula.

Similarly, using the above method, we can calculate the speed at which a spacecraft flies out of the solar system, also known as the third cosmic velocity. From equation (11), we can roughly calculate the speed at which the Earth orbits the Sun

$$\upsilon = \sqrt{\frac{GM_\odot}{R}} = \sqrt{\frac{1.327\times 10^{20}}{1.496\times 10^{11}}} = 2.978\times 10^4 \text{ (m/s)} \quad (30)$$

If an artificial planet with the same radius of revolution as the Earth is launched from a stationary state relative to the solar system, according to the work-kinetic energy theorem, an additional factor must be added to the Earth's revolution speed. Namely

$$v = \sqrt{2}\upsilon = \sqrt{2}\times 2.978\times 10^4 m/s = 4.212\times 10^4 m/s \quad (31)$$

The speed increment caused by factor (√2) is

$$\Delta v = v - \upsilon = (4.212 - 2.978)\times 10^4 m/s = 1.234\times 10^4 m/s \quad (32)$$

If this artificial planet is launched from Earth and is launched eastward from near the equator, the required speed is the lowest. At this point, two factors must be considered for their impact. One is the second cosmic velocity required to overcome the gravitational pull of the Earth, which is 11.18 km/s. The second cosmic velocity already includes the velocity increment caused by the √2 factor. The other is the speed increment caused by the √2 factor added to the Earth's revolution speed, which is 12.34 km/s. Therefore, the speed that needs to be reached when calculated by the work-kinetic energy theorem is

$$v_{III} = \sqrt{\Delta v^2 + v_{II}^2} = \sqrt{(1.234)^2 + (1.118)^2}\times 10^4 = 1.665\times 10^4 \text{ (m/s)} \quad (33)$$

The equation (33) yields the third cosmic velocity of 16.7 km/s. The second and third cosmic velocities are both referred to as escape velocities. Due to incorrect understanding and use of the kinetic energy formula in current theory, a factor of √2 must be added when calculating the escape velocity.

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## Summary of the relationship between work and kinetic energy

Due to the current theory not distinguishing between instantaneous kinetic energy and average kinetic energy, the understanding of work and kinetic energy has become confusing. Based on the two necessary conditions for doing work (force $F$ and distance l), the complex relationship between work and kinetic energy is summarized as follows:

1. A uniformly moving object does not do work because it has a distance but no force ($l \neq 0$, $F = 0$, $\therefore W = 0$), and its kinetic energy is $E = mv^2$.
2. The concept of kinetic energy of a variable speed moving object can be divided into average kinetic energy of the process and instantaneous kinetic energy. The instantaneous kinetic energy describes the energy state of a moving object, while the average kinetic energy describes the change in energy state. The instantaneous kinetic energy does not do work because it has force but has no distance (no duration for displacement), that is, $F \neq 0$, $l = 0$, $\Delta t = 0$, $\therefore W = 0$. The instantaneous kinetic energy is $E = mv^2$.
3. An object in variable speed motion has an average kinetic energy doing work, because it has force and has a distance (has a duration for displacement), that is, $F \neq 0$, $l \neq 0$, $\Delta t \neq 0$, $\therefore W \neq 0$. Work and average kinetic energy can be converted into each other, that is, $W = \frac{1}{2}mv^2$. The average kinetic energy is given by the current kinetic energy formula, $EK = \frac{1}{2}mv^2$. This kinetic energy (EK) is the average kinetic energy, and is also the instantaneous kinetic energy when the velocity is equal to the root mean square velocity $v/\sqrt{2}$.
4. Object undergoes uniform deceleration motion from a constant speed $\upsilon$ as the starting point. ① At the starting point, $v = \upsilon$. The instantaneous kinetic energy is $m\upsilon^2$ or $mv^2$; At the end point (rest), the instantaneous kinetic energy decreases to $0$. During the process the average kinetic energy does work $W = \frac{1}{2}mv^2 = \frac{1}{2}m\upsilon^2$. ② According to the current theoretical explanation for the kinetic energy formula, the kinetic energy of an object at velocity $v$ is $\frac{1}{2}mv^2$, all kinetic energy does work $\frac{1}{2}mv^2$. The first misunderstanding is that the kinetic energy at velocity ($v$) is $\frac{1}{2}mv^2$, second misunderstanding is that all kinetic energy ($\frac{1}{2}mv^2$) does

work (½$mv^2$). The two misunderstandings offset each other, resulting in consistent results with the experiment. This may be the reason why people misunderstand the work-kinetic energy theorem.

5. An object accelerates from rest. ① doing work (½$mv^2$) on the object during the process, by the conservation of energy, the average kinetic energy of the object during the process reaches (½$mv^2$). At the endpoint, the object only reached the root mean square velocity ($v/\sqrt{2}$). The doing work can only be converted into average kinetic energy (½$mv^2$), and it is only equivalent to the instantaneous kinetic energy of the object at its root mean square velocity ($v/\sqrt{2}$). ② According to the current theoretical explanation for the kinetic energy formula, if an object is uniformly accelerated to velocity $v$, during which work (½$mv^2$) is done on the object, all converted into the kinetic energy of the object at the velocity v, and the kinetic energy of the object is (½$mv^2$). This kinetic energy has been proven to be too small at high speeds, but in reality, objects only gain average kinetic energy (½$mv^2$). At the end of the process, the object did not reach the velocity $v$, but only reached the root mean square velocity ($v/\sqrt{2}$). The root mean square velocity is equal to a constant velocity, i.e. $v/\sqrt{2} = \upsilon$. To achieve a uniform speed $\upsilon$, by current theories the speed $\upsilon$ must be multiplied by a factor √2, that is, $v=\sqrt{2}\ \upsilon$.

All celestial bodies in the universe are in rotational motion, and the centrifugal kinetic energy of motion is equal to the gravitational potential energy. We prove that centrifugal kinetic energy is the kinetic energy of uniform motion. All experiments at high speeds have proven that the kinetic energy $mv^2$ is correctness. This work will achieve the unification of the kinetic energy formula from low speed to high speed, and will complete the unification of the kinetic energy formula and the mass energy relationship formula.

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## Author Contributions Statement

Xiao En Wang devised the project, analysed the data, designed, drafted the manuscript and contributed to all of the final version of this work.

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